Coulomb

Charles-Augustin de Coulomb (14 June 1736 – 23 August 1806) was a French physicist. He is best known for developing Coulomb's law, the definition of the electrostatic force of attraction and repulsion. The SI unit of electric charge, the coulomb, was named after him. He was involved in engineering, in structural, fortifications, soil mechanics, as well as other fields of engineering.

http://en.wikipedia.org/wiki/Charles-Augustin_de_Coulomb

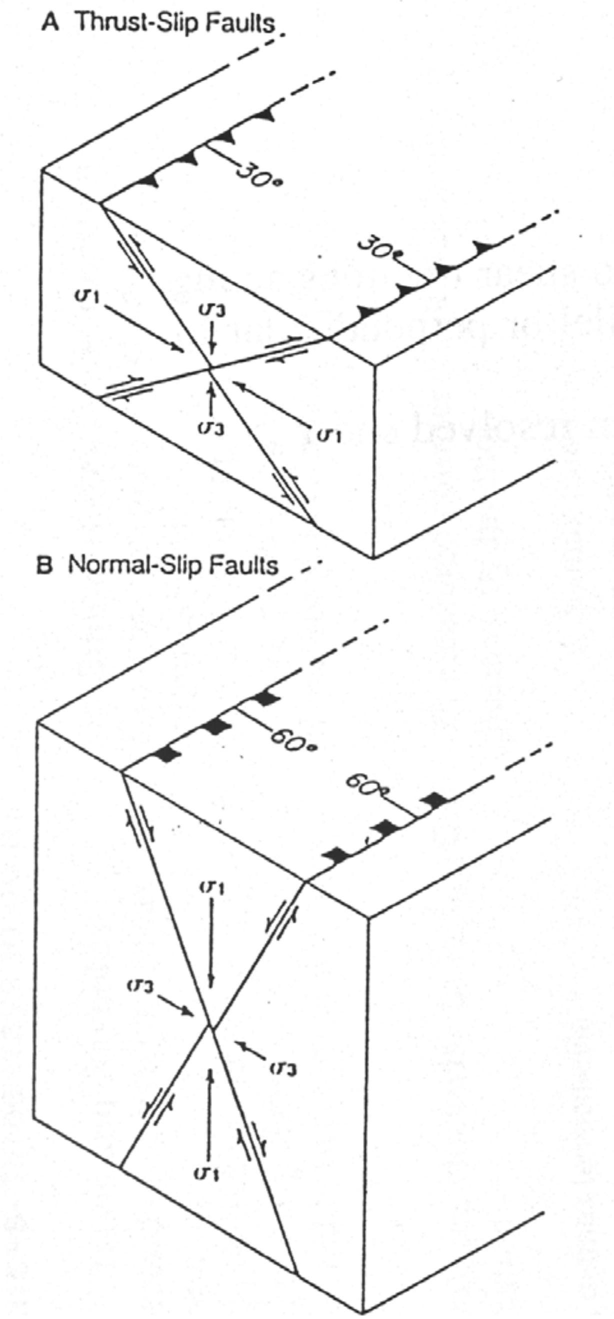
Tractions and stress

- A more strict definition of stress
- Traction is stress relative to a surface through a point **p**.
- Stress tensor is the field of tractions acting over a point **p**.
- Stress field is the entire collection of stress tensors in a body.

Andersonian Faulting Theory

- Key assumptions:
  - Earth's surface is a free surface (so it has no shear tractions acting along it). Therefore, σ_1, σ_2, σ_3 must be either parallel or perpendicular to it.
  - A fault will slip in the direction of maximum resolved shear traction

Andersonian Faulting Theory



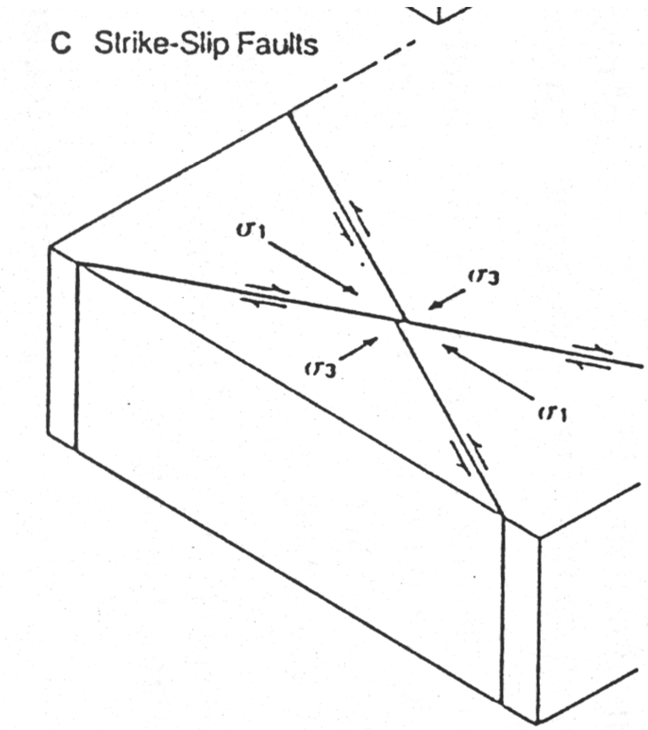


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Figure 6.62 Schematic representation of (*A*) thrust faults, (*B*) normal faults, and (*C*) strike–slip faults at or near the surface of the Earth. These are the likely orientations since each of the three principal stress directions at or near the surface of the Earth is either horizontal or vertical, and since the angle of internal friction for rocks is almost always approximately 30°.

Davis and Reynolds, p. 306

The Stress Equations

Thankfully, once the magnitudes and orientations of the principal stresses at a point are known, we can readily calculate the normal stress (σ_n) and shear stress (σ_s) for planes of any orientation using the **fundamental stress equations** derived in standard engineering and structural geological texts (e.g., Ramsay, 1967; Jaeger and Cook, 1976; Means, 1976).

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta$$

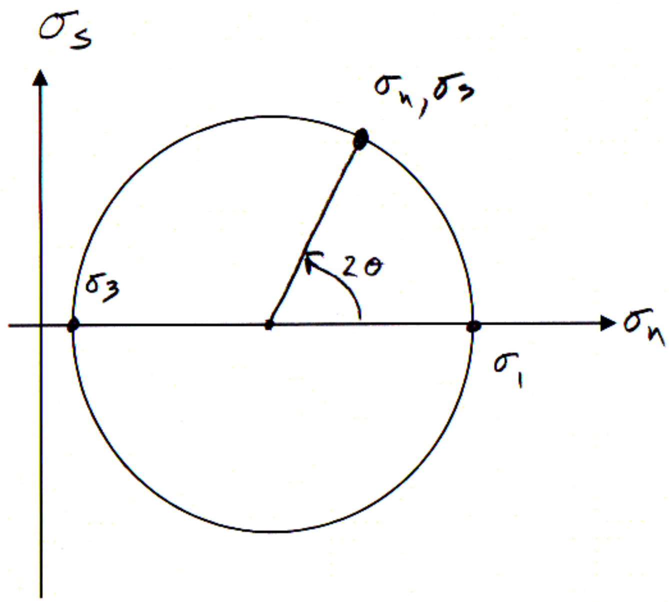
$$\tau = \sigma_s = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta$$

Faults, stress, and tractions

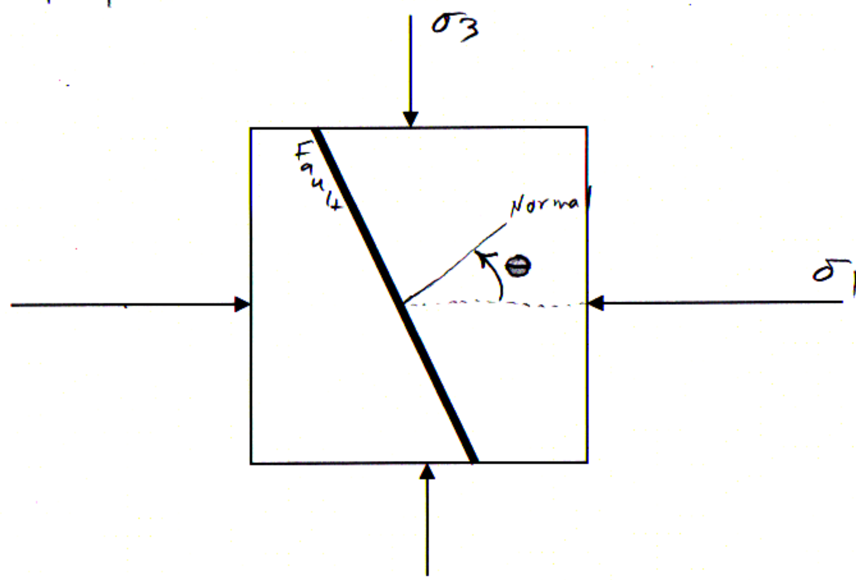
Mohr Circle

Graphical construction that lets us visualize the relationship between the principal stresses and tractions on a boundary (like a fault).

Mohr Space

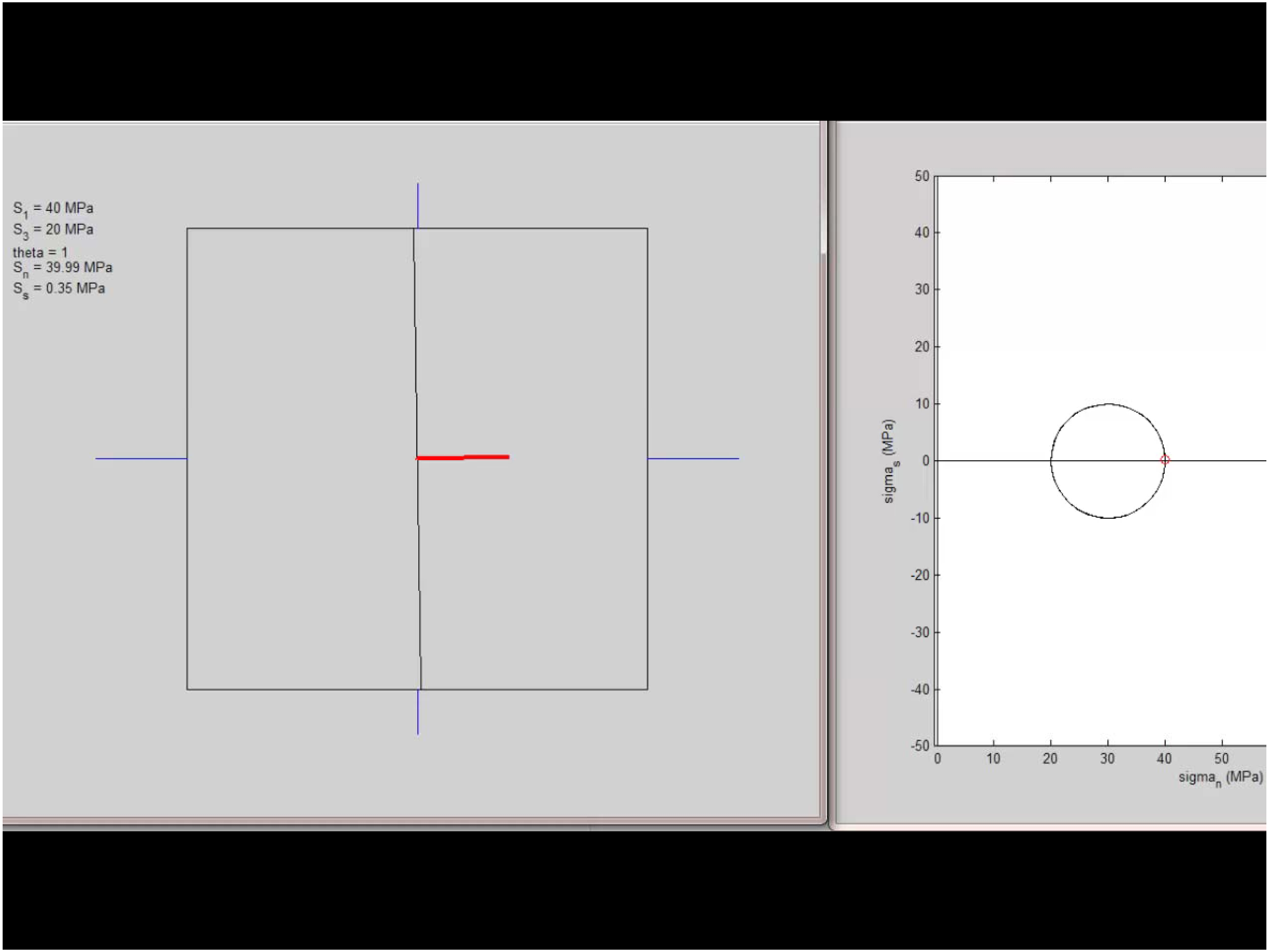


Physical space



θ measured positive counter clockwise from σ_1 direction to normal of plane of interest

2θ measure positive counter clockwise from σ_n direction on Mohr circle



S1 = 40 MPa
S3 = 20 MPa
theta = 1
Sn = 39.99 MPa
Ss = 0.35 MPa
sigma_s (MPa)
sigma_n (MPa)

Reprinted from Nature, Vol. 341, No. 6240, pp. 291–298, 28th September, 1989

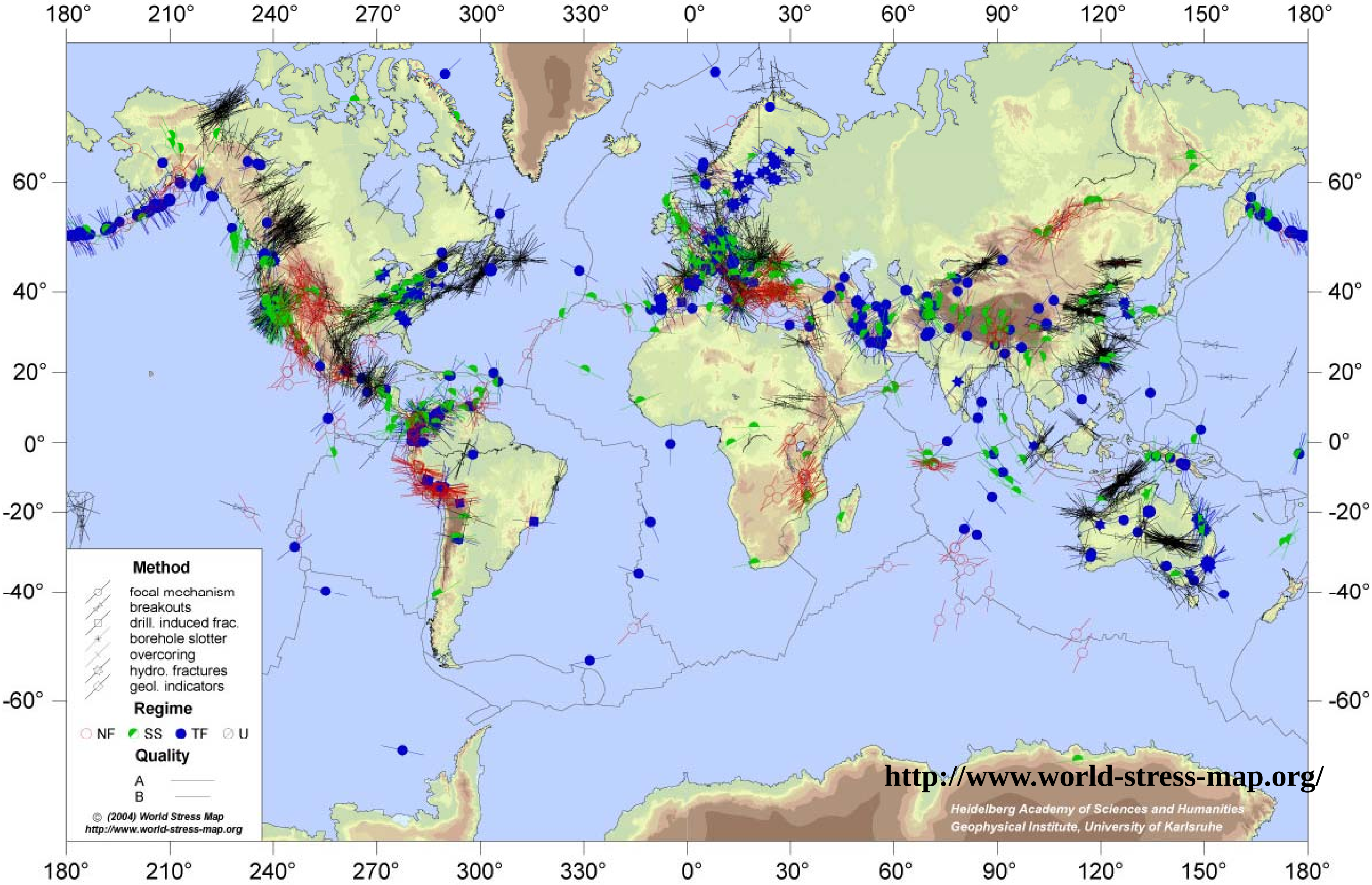
Global patterns of tectonic stress

Mary Lou Zoback, Mark D. Zoback, J. Adams, M. Assumpção, S. Bell, E. A. Bergman, P. Blümling, N. R. Brereton, D. Denham, J. Ding, K. Fuchs, N. Gay, S. Gregersen, H. K. Gupta, A. Gvishiani, K. Jacob, R. Klein, P. Knoll, M. Magee, J. L. Mercier, B. C. Müller, C. Paquin, K. Rajendran, O. Stephansson, G. Suarez, M. Suter, A. Udias, Z. H. Xu & M. Zhizhin

---

Regional patterns of present-day tectonic stress can be used to evaluate the forces acting on the lithosphere and to investigate intraplate seismicity. Most intraplate regions are characterized by a compressional stress regime; extension is limited almost entirely to thermally uplifted regions. In several plates the maximum horizontal stress is subparallel to the direction of absolute plate motion, suggesting that the forces driving the plates also dominate the stress distribution in the plate interior.

World Stress Map

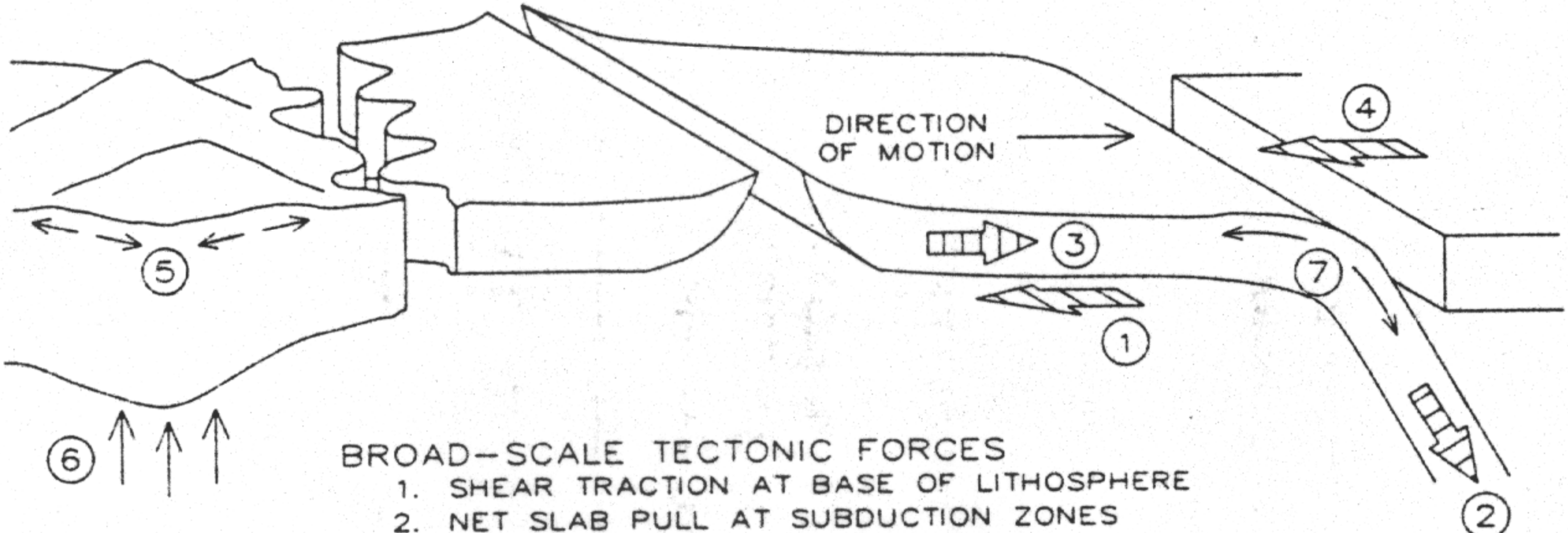


http://www.world-stress-map.org/

http://www.world-stress-map.org/

North America Stress Map

SOURCES OF TECTONIC STRESS



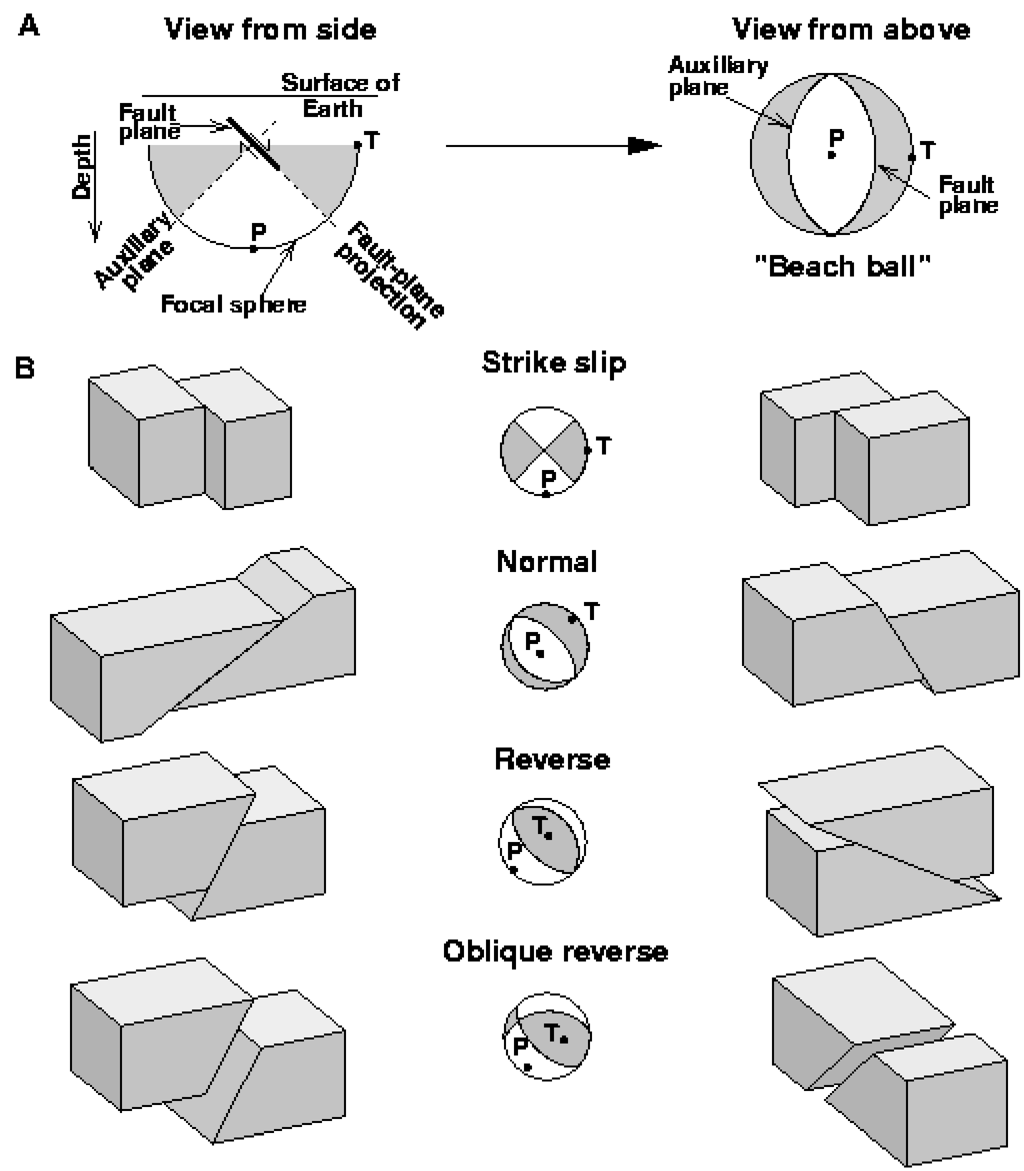
BROAD-SCALE TECTONIC FORCES

1. SHEAR TRACTION AT BASE OF LITHOSPHERE
2. NET SLAB PULL AT SUBDUCTION ZONES
3. RIDGE PUSH FROM OCEANIC RIDGES
4. TRENCH SUCTION ON OVER-RIDING PLATE

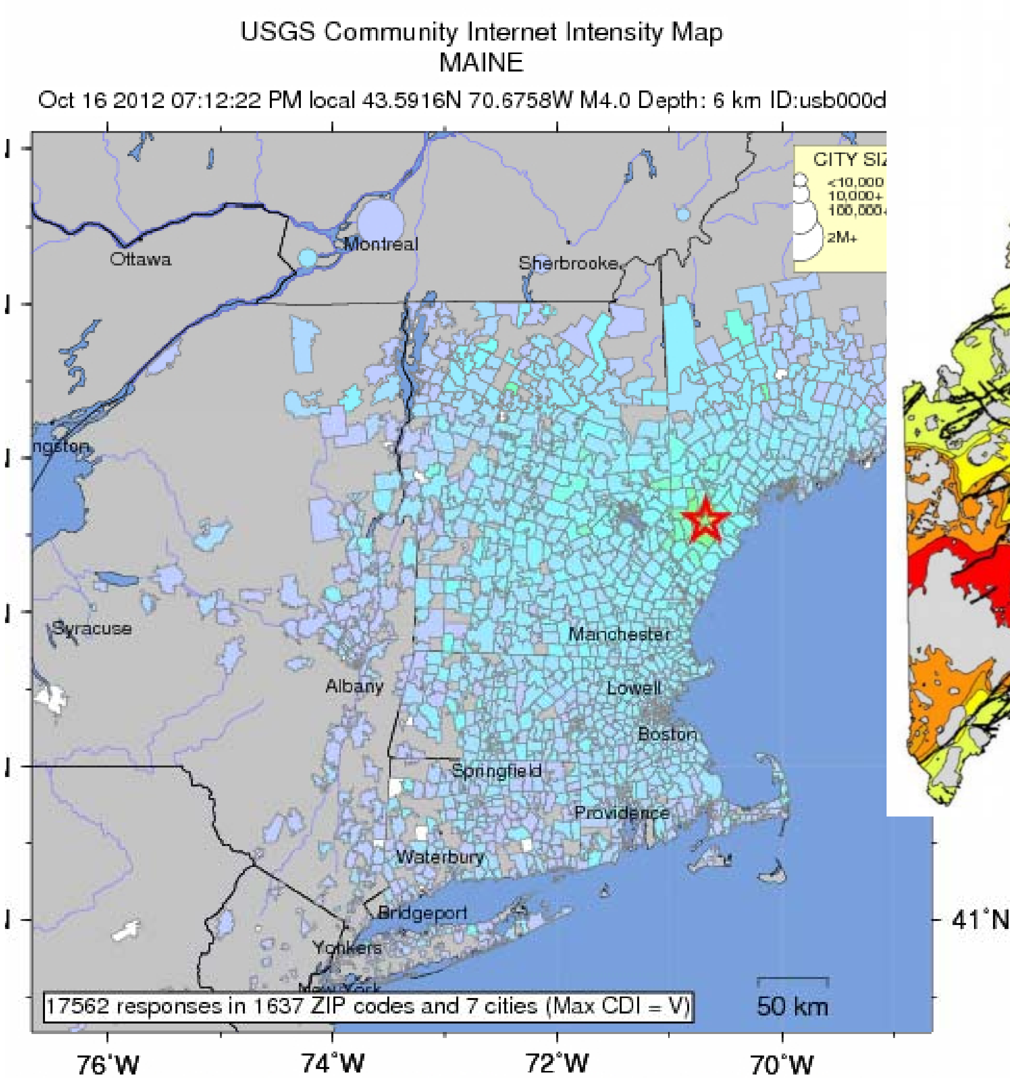
LOCAL TECTONIC STRESSES

5. BENDING DUE TO SURFACE LOADS
6. ISOSTATIC COMPENSATION
7. DOWNBENDING OF OCEANIC LITHOSPHERE

Schematic diagram of a focal mechanism

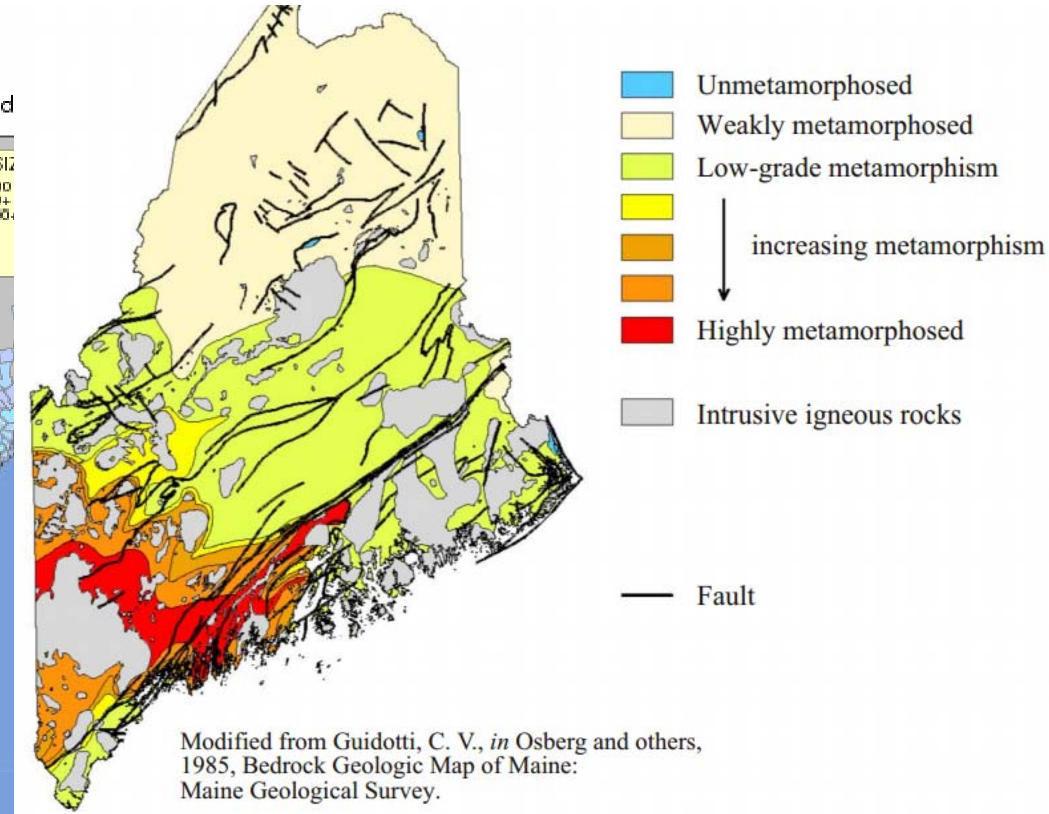


USGS, 1996

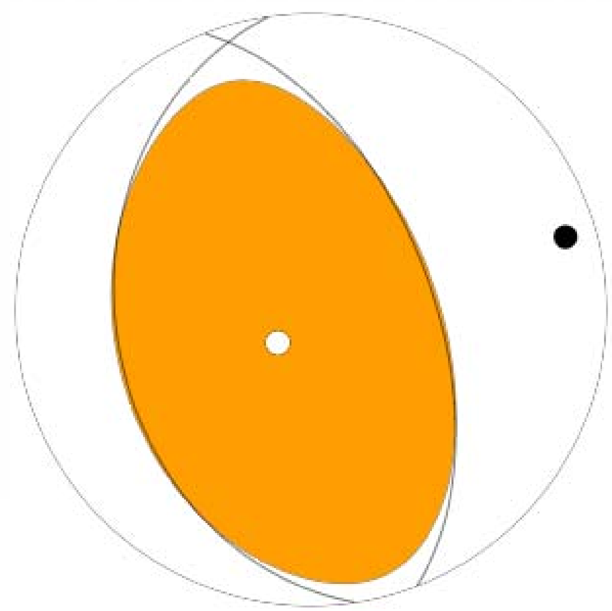


INTENSITY	I	II-III	IV	V	VI	VII	VIII	IX	X+
SHAKING	Not felt	Weak	Light	Moderate	Strong	Very strong	Severe	Violent	Extreme
DAMAGE	none	none	none	Very light	Light	Moderate	Moderate/Heavy	Heavy	V. Heavy

Processed: Wed Oct 17 15:27:58 2012



Modified from Guidotti, C. V., *in* Osberg and others, 1985, Bedrock Geologic Map of Maine: Maine Geological Survey.



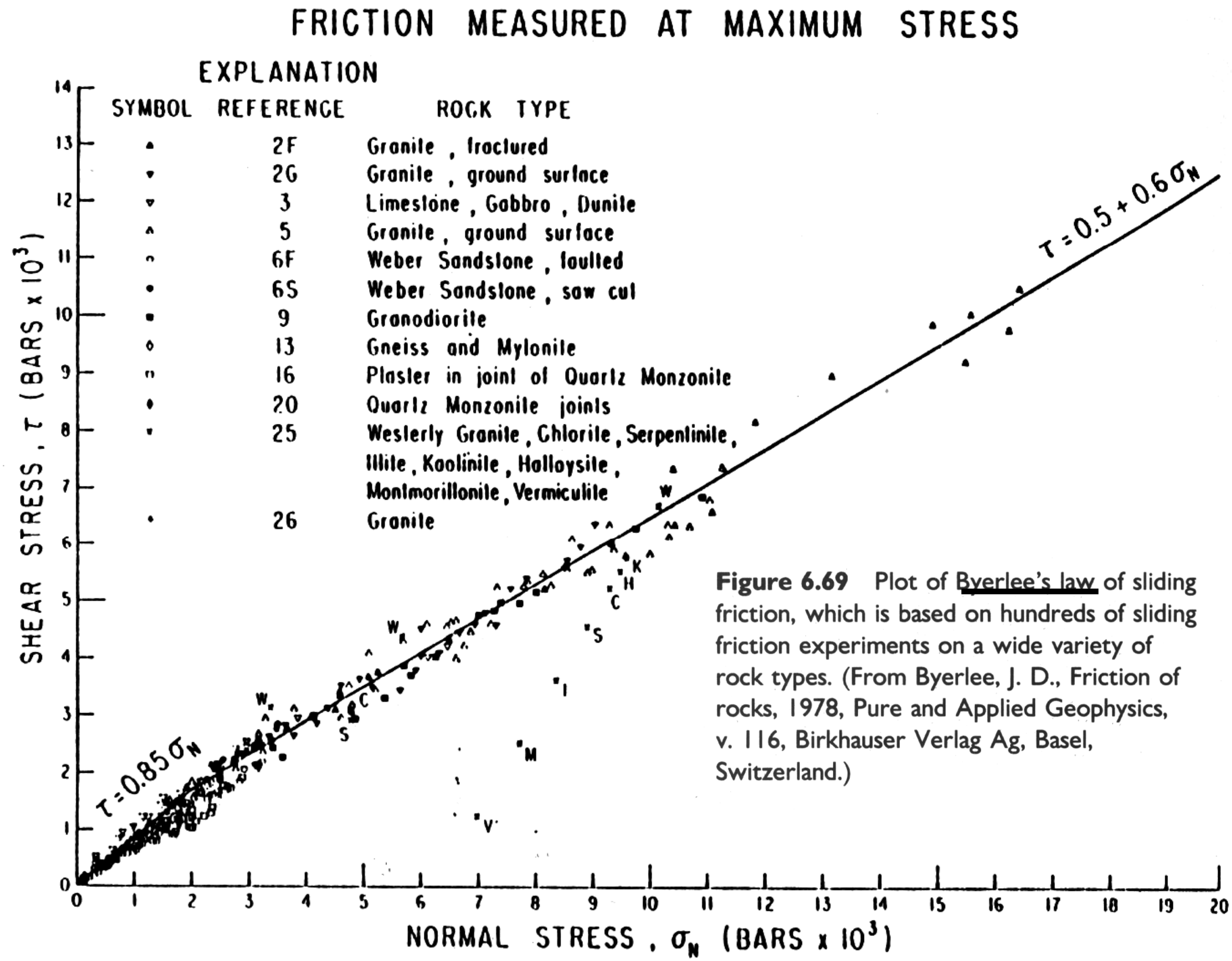
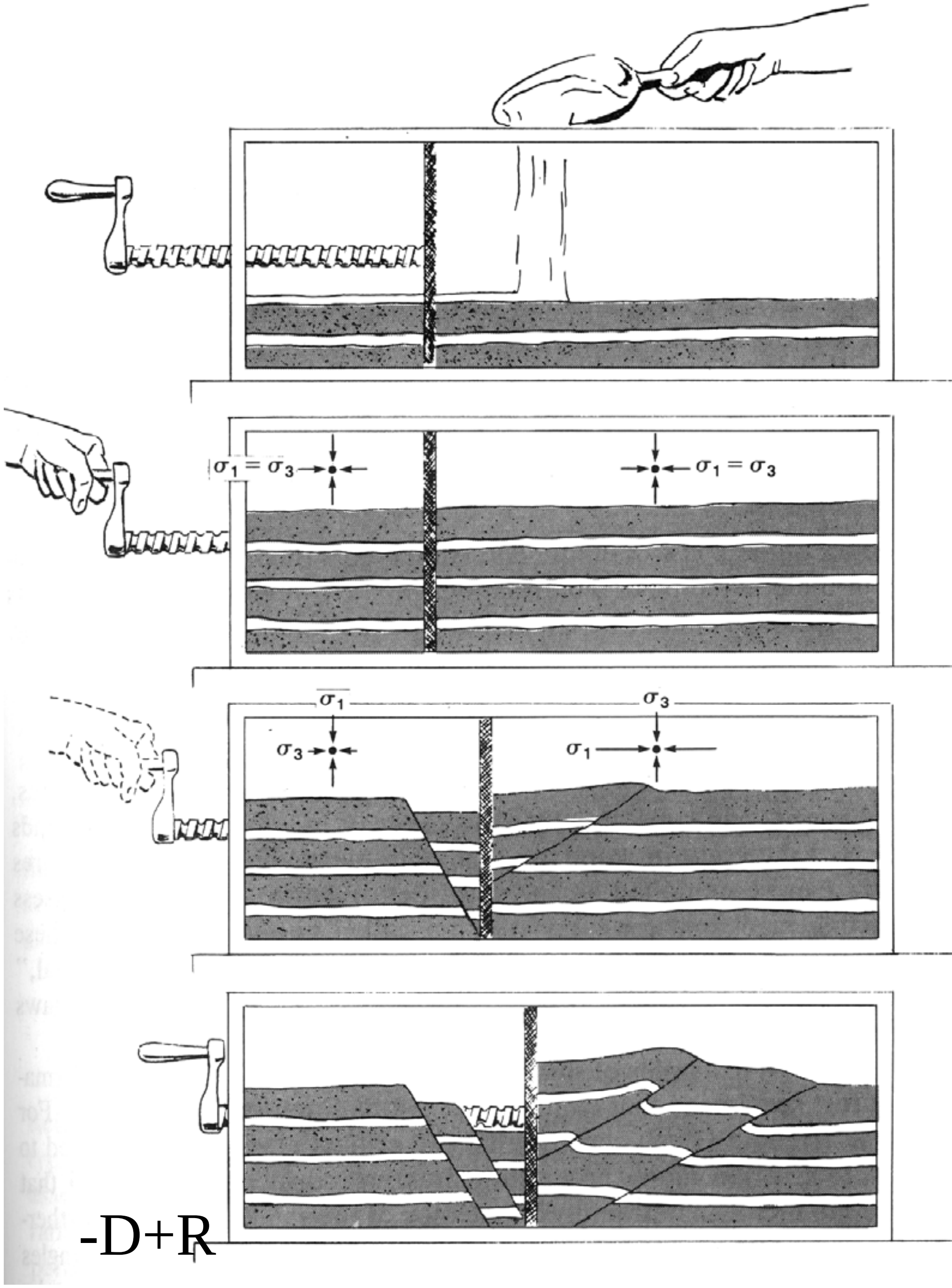


Figure 6.69 Plot of Byerlee's law of sliding friction, which is based on hundreds of sliding friction experiments on a wide variety of rock types. (From Byerlee, J. D., Friction of rocks, 1978, Pure and Applied Geophysics, v. 116, Birkhauser Verlag Ag, Basel, Switzerland.)



-D+R

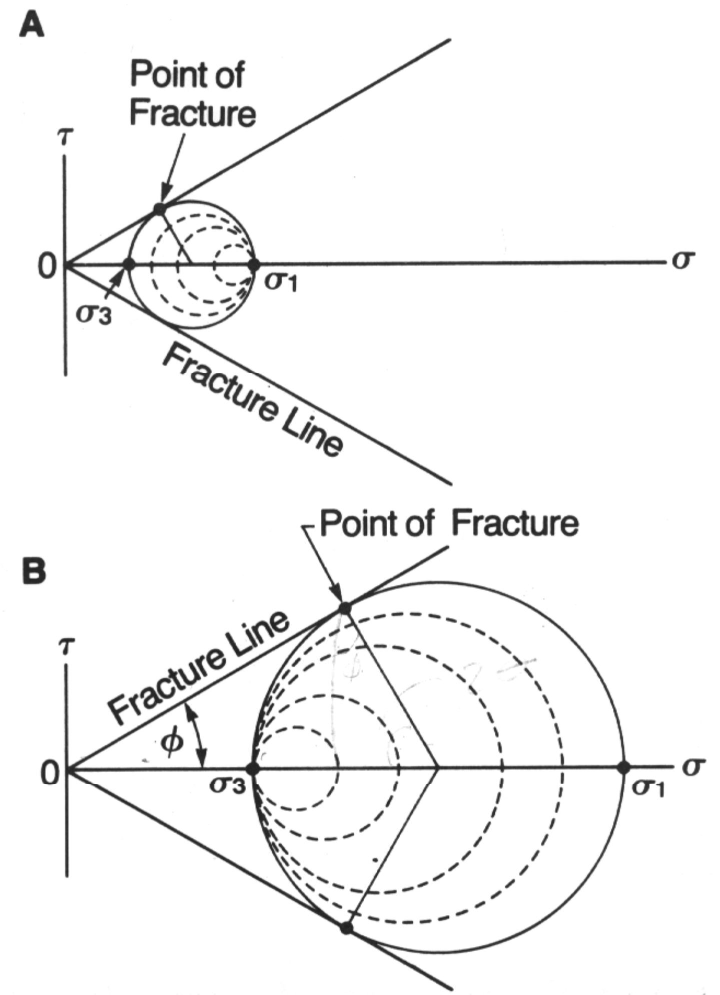


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Figure 6.67 Mohr diagram portrayal of the dynamic conditions of the sandbox experiment. (*A*) Differential stress conditions leading to normal faulting in the left-hand compartment. (*B*) Differential stress conditions leading to thrust faulting in the right-hand compartment.

Geophys. J. Int. (1997) **128,** 594–604

Progressive failure on the North Anatolian fault since 1939 by earthquake stress triggering

Ross S. Stein,[1] Aykut A. Barka[2] and James H. Dieterich[1]

[1] *US Geological Survey,* MS 977, *Menlo Park,* CA 94025 *USA. E-mail: rstein@usgs.gov; jdieterich@usgs.gov*

[2] *Istanbul Technical University, Department of Geology, Istanbul,* 80626 *Turkey. E-mail: barka@sariyer.cc.itu.edu.tr*

Since we lack hypocentral depths for the Anatolian main shocks, we sample stress in the central part of the seismogenic crust at a depth of 8 km. Failure is facilitated on a plane when the Coulomb failure stress σ_f rises,

$$\sigma_f = \tau_\beta - \mu\sigma'_\beta, \tag{1}$$

where τ_β is the shear stress on the failure plane β (positive in the direction of fault slip) and σ'_β is the effective normal stress (positive in compression); $\mu\sigma'_\beta \approx \mu'\sigma_\beta$, where μ' is the apparent coefficient of friction with range 0.0–0.75. The confining stress can be related to fluid pore pressure by Skempton's coefficient, B, the ratio of the change in pore pressure in a cavity to the change in applied stress, where $\mu' = \mu(1 - B)$ (Roeloffs 1988). Immediately after the earthquake, $B \approx 2/3$, but could fall to zero if fluids were to drain fully from the fault zone (Scholz 1990). We set $\mu' = 0.4$, equivalent to laboratory values of friction ($\mu \sim 0.75$) and moderate pore pressure if fluids are not fully expelled ($B \sim 0.5$), or to a low value of friction as inferred for the San Andreas fault (Zoback *et al.* 1987). A μ' of 0.4 also minimizes the calculation error caused by the uncertainty in μ' to ± 25 per cent (King, Stein & Lin 1994); compare, for example, column 2 (for $\mu' = 0.75$) and column 3 (for $\mu' = 0.40$) in Table 2.

Summary of 3D stress resolution

Ramón Arrowsmith
Thanks to Don Ragan and Olaf Zielke

Given **S** principal stress tensor with orientation $x'y'z'$

Rotate to N-S, E-W components

$$\mathbf{R} = \begin{bmatrix} l & l' & l'' \\ m & m' & m'' \\ n & n' & n'' \end{bmatrix} \text{ where } \begin{array}{rl} l & = x * x' \\ l' & = x * y' \\ l'' & = x * z' \\ \dots & \end{array}$$

$$\mathbf{S'} = \mathbf{R}^T\mathbf{S}\mathbf{R}$$

And given plane with normal vector direction cosines **N**

Traction $\mathbf{T} = \mathbf{S'} * \mathbf{N}$ (row and column multiplication)
$T = \sqrt{\mathbf{T}(1)^2 + \mathbf{T}(2)^2 + \mathbf{T}(3)^2}$ traction magnitude

$\sigma_n = \mathbf{T} \cdot \mathbf{N}$ dot product for normal traction magnitude

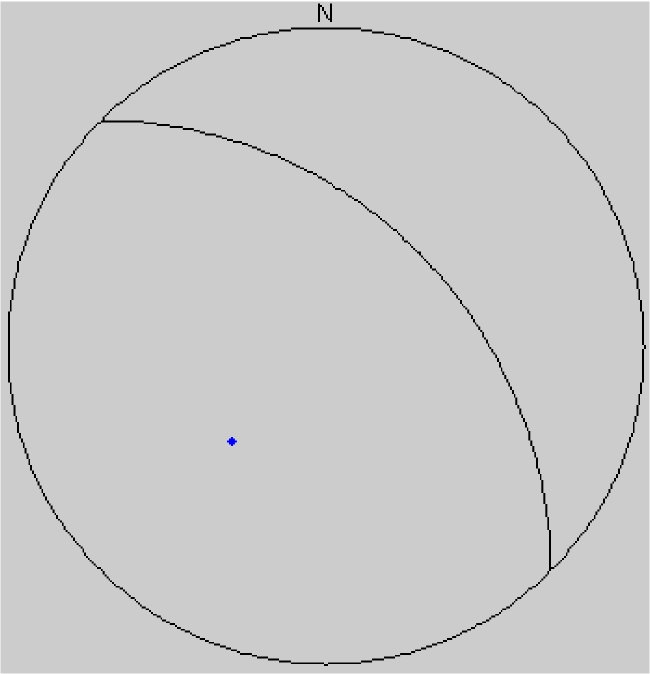
$\mathbf{B} = \mathbf{T} \times \mathbf{N}$ cross product for null vector
$B = \sqrt{\mathbf{B}(1)^2 + \mathbf{B}(2)^2 + \mathbf{B}(3)^2}$ **B** magnitude
$\mathbf{B}_{normalized} = \mathbf{B}./B$ normalize for orientation if necessary

$\mathbf{T_s} = \mathbf{N} \times \mathbf{B}$ cross product for shear traction vector
$\tau = \sqrt{\mathbf{T_s}(1)^2 + \mathbf{T_s}(2)^2 + \mathbf{T_s}(3)^2}$ shear traction magnitude
$\mathbf{T_s}_{normalized} = \mathbf{T_s}./\tau$ normalize for shear traction orientation

Coulomb failure function: $\Delta\sigma_f = \Delta\tau - (\mu - P)\Delta\sigma_n$

Example application to South Mountains faults

```
%Set up Receiver Fault
%We are interested in a plane with P(75,225)
poleplunge=45;
poletrend =225;

[l,m,n]
=plunge_trend_to_dir_cosines(poleplunge,poletrend);
ld1 = -l; md1 = -m; nd1 = cosd(poleplunge);
[dip, dipdir] = dir_cosines_to_plunge_trend(ld1, md1,
nd1);


N=[l;m;n];
```



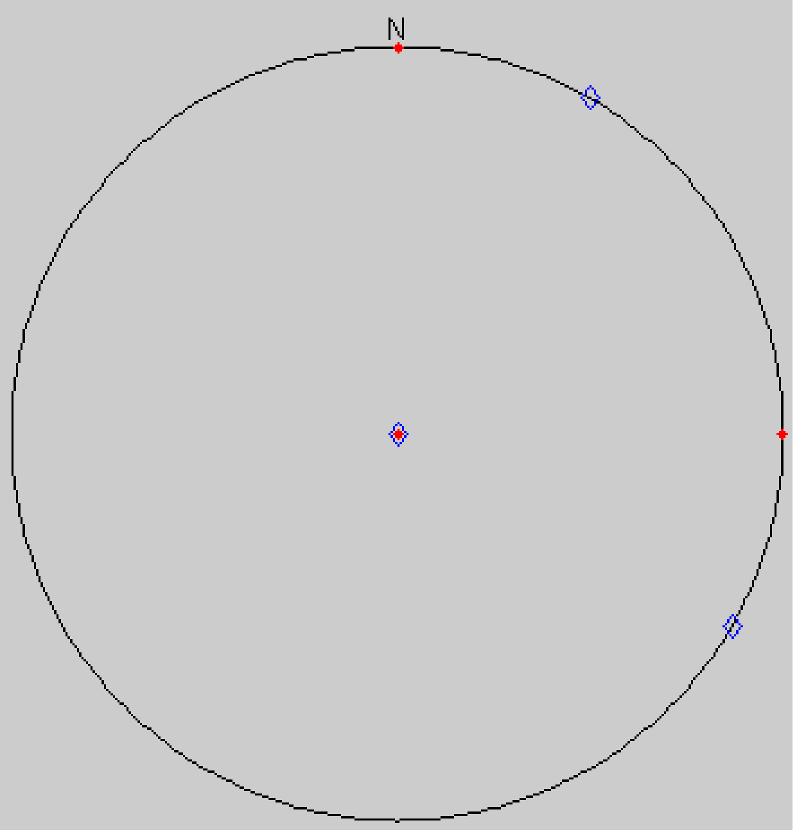
dip is 45.0 and dip dir is 45.0

Example application to South Mountains faults

```
%Set up stress tensor
%assume that the principal stresses are appropriate for normal faulting
%conditions so maximum stress is the vertical stress
sv = -26.7.*12; %assume 26.5 MPa per km and 12 km depth
shmin = sv.*0.1; %assume the 1 direction is the minimum horizontal stress and is 10%
shmax = sv.*0.25; %assume the 2 direction is intermediate
S = [shmin 0 0;
         0 shmax 0;
         0   0 sv]
%buildrotationmatrix2(xprimetrend, xprimeplunge, yprimetrend,yprimeplunge,zprimetrend,zprimeplunge, talkandplot)
 R = buildrotationmatrix2(      30,            0,           120,             0,            0,           90, 1)
```

rotatedS = R'*S*R

```
S =

  -32.0400         0         0
         0  -80.1000         0
         0         0 -320.4000

xprime l = 0.8660 m = 0.5000 n = 0.0000
yprime l = -0.5000 m = 0.8660 n = 0.0000
zprime l = -0.0000 m = -0.0000 n = 1.0000
checks for orthogonality: xy 0.0000 xz 0.0000 yz 0.0000

R =

    0.8660   -0.5000         0
    0.5000    0.8660         0
         0         0    1.0000


rotatedS =

  -44.0550  -20.8106         0
  -20.8106  -68.0850         0
         0         0 -320.4000
```



Example application to South Mountains faults

```
%Now resolve the stresses
T=rotatedS*N; %equation 13.11

T_mag = sqrt(sum(T.^2));

%normalize components of T to get its direction cosines
lt=T(1)./T_mag; mt = T(2)./T_mag; nt = T(3)./T_mag;

%plot traction vector
[plunge, trend] = dir_cosines_to_plunge_trend2(lt, mt, nt);

%we know the orientation of the normal traction,
%but what is its magnitude?
sigma = dot(T,N); %equation 13.13
```

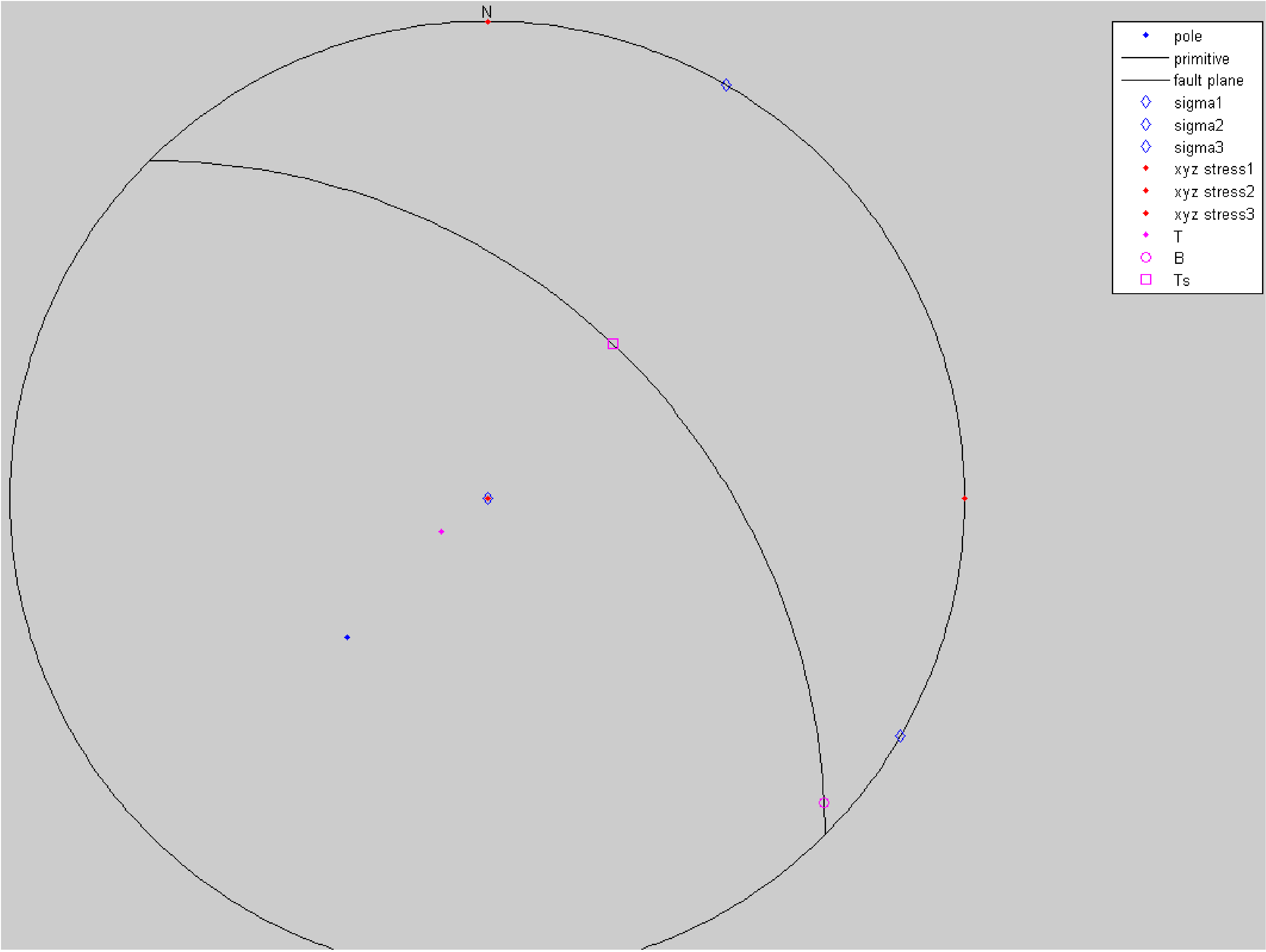
traction vector components are 32.4328 44.4478 -226.5570
traction magnitude 233.1428
traction vector direction cosines 0.1391 0.1906 -0.9718
traction plunge = 76.3 trend = 233.9
normal traction mag -198.64

Example application to South Mountains faults

```
%Now for the shear traction; use the McKenzie construction
B = cross(T,N); %vector normal to the plane containing T and N
B_mag = sqrt(B(1)^2 + B(2)^2 + B(3)^2);
lb = B(1)./B_mag;
mb = B(2)./B_mag;
nb = B(3)./B_mag;

[plunge, trend] = dir_cosines_to_plunge_trend2(lb,mb,nb);

Ts = cross(N,B); %shear traction direction
Ts_mag = sqrt(Ts(1)^2 + Ts(2)^2 + Ts(3)^2);
Ts(1) = Ts(1)./Ts_mag;
Ts(2) = Ts(2)./Ts_mag;
Ts(3) = Ts(3)./Ts_mag;
```

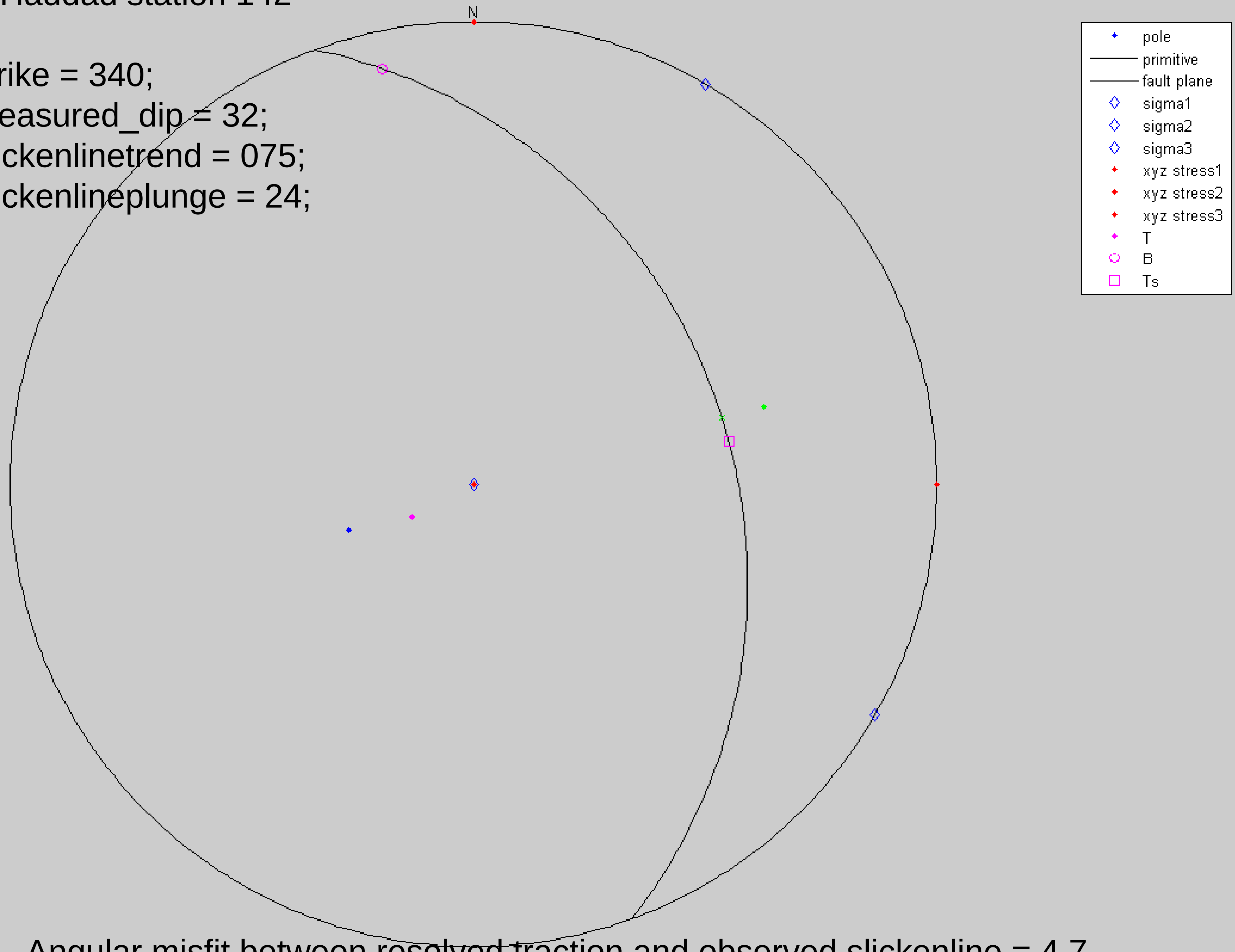
shear traction mag 122.06
check that components make
same length as traction: 233.1428 =?= 233.1428

```
[plunge, trend] = dir_cosines_to_plunge_trend2(Ts(1), Ts(2),
Ts(3));

%let's check that the normal and shear are components of the
traction
testmag = sqrt(sum(sigma.^2 + Ts_mag.^2));
```

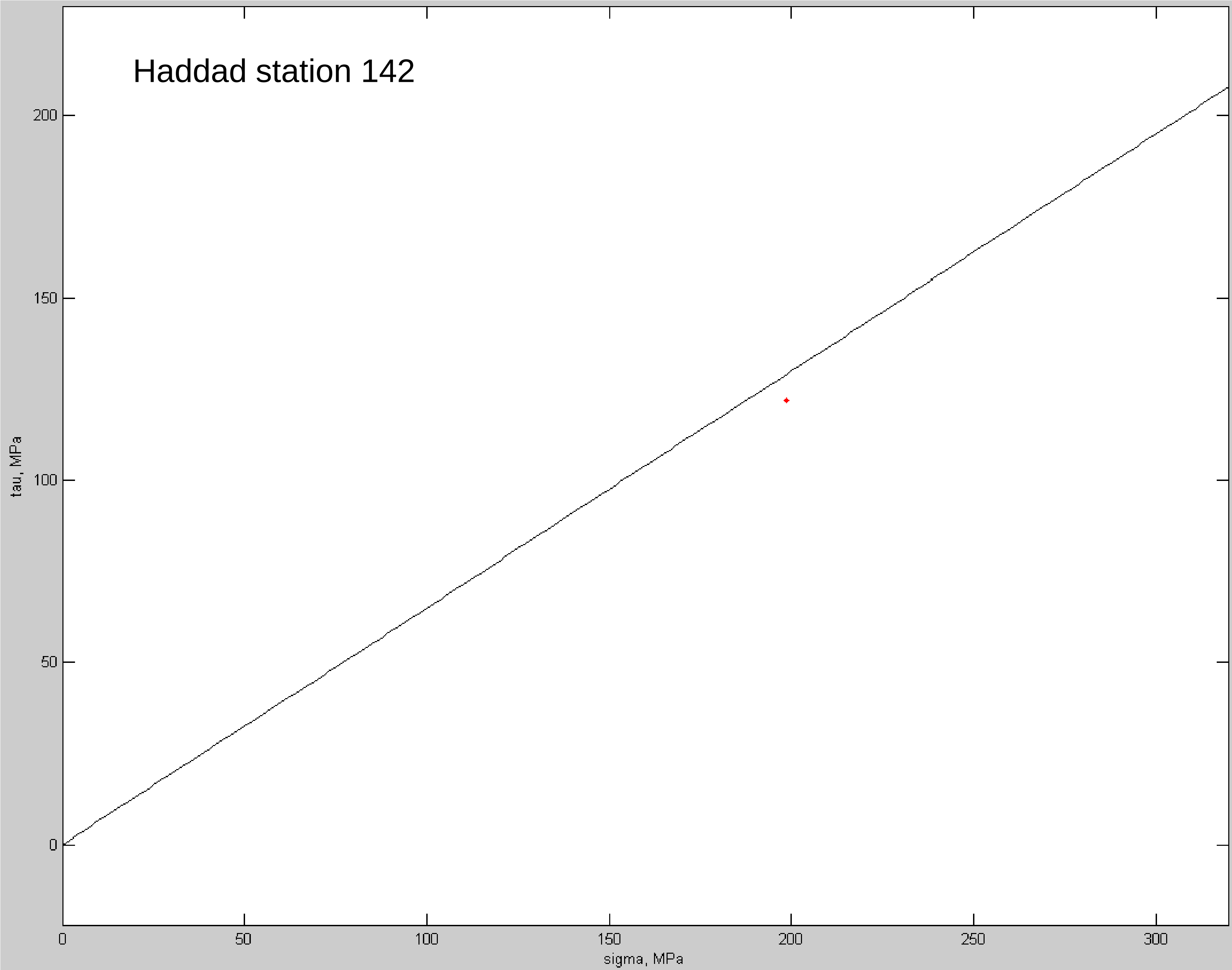

N
pole
primitive
fault plane
sigma1
sigma2
sigma3
xyz stress1
xyz stress2
xyz stress3
T
B
Ts

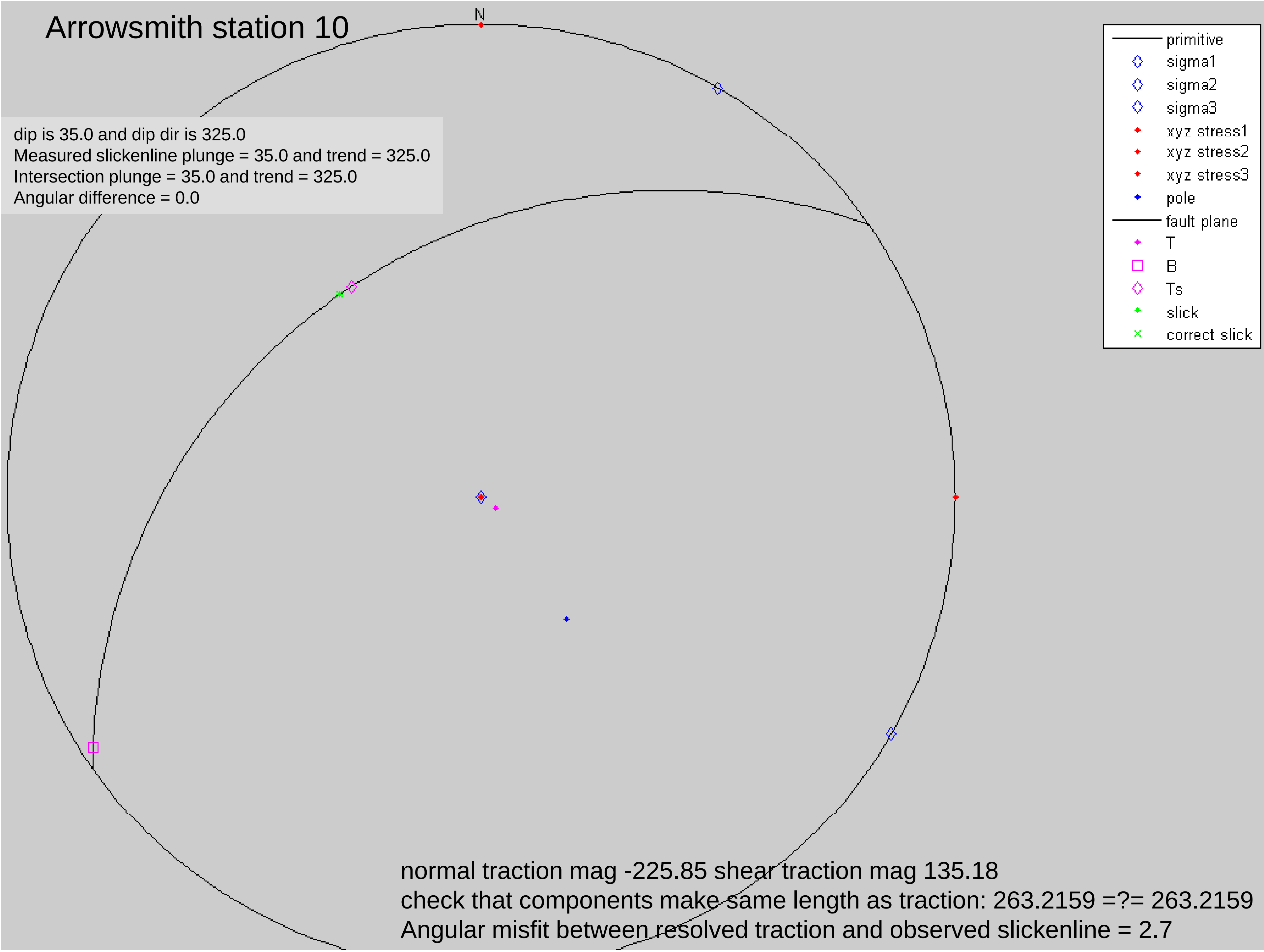
%Haddad station 142

strike = 340;
measured_dip = 32;
slickenlinetrend = 075;
slickenlineplunge = 24;

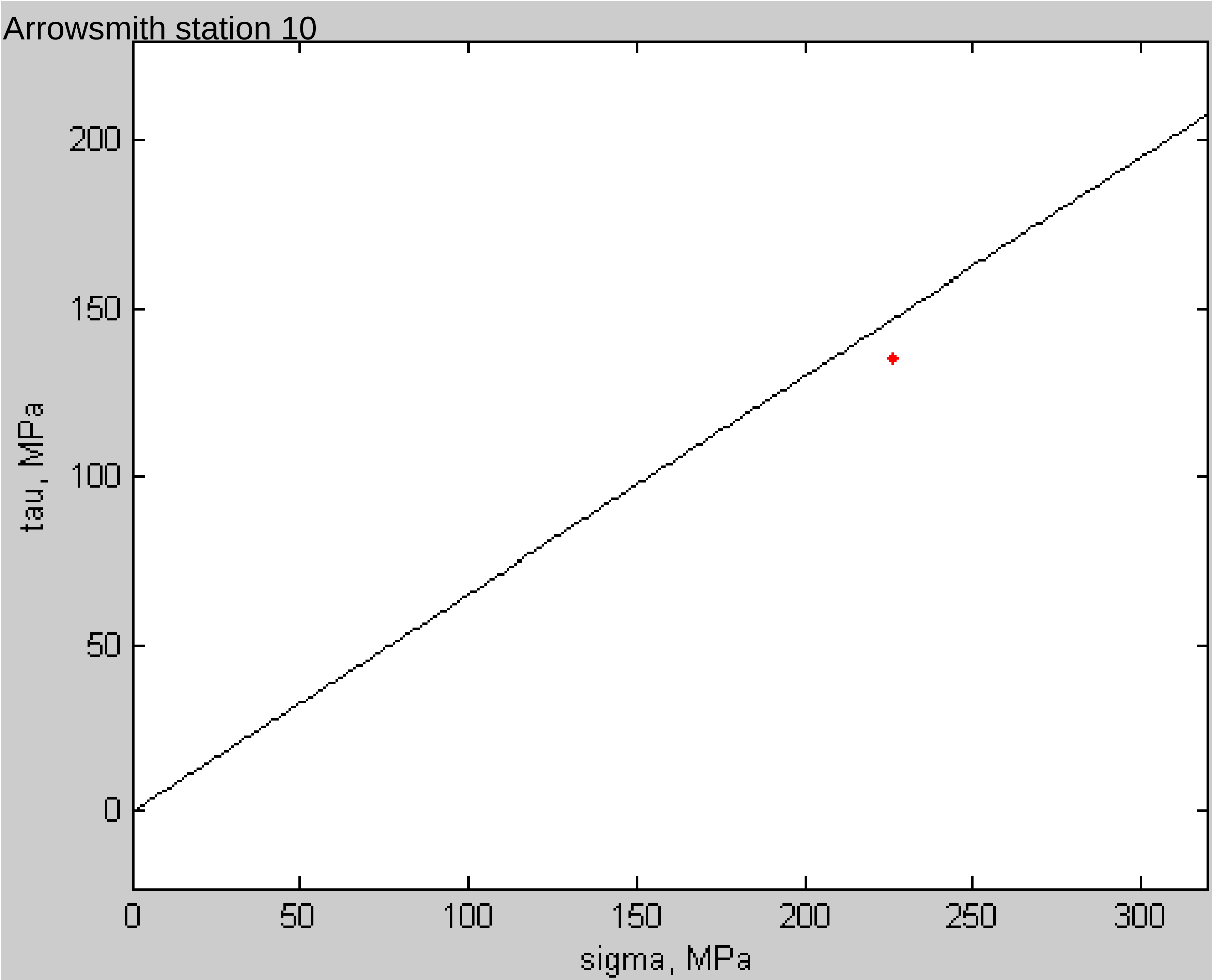


Angular misfit between resolved traction and observed slickenline = 4.7


Haddad station 142
tau, MPa
sigma, MPa
0
50
100
150
200
250
300
0
50
100
150
200



Arrowsmith station 10
N
dip is 35.0 and dip dir is 325.0
Measured slickenline plunge = 35.0 and trend = 325.0
Intersection plunge = 35.0 and trend = 325.0
Angular difference = 0.0
primitive
sigma1
sigma2
sigma3
xyz stress1
xyz stress2
xyz stress3
pole
fault plane
T
B
Ts
slick
correct slick
normal traction mag -225.85 shear traction mag 135.18
check that components make same length as traction: 263.2159 =?= 263.2159
Angular misfit between resolved traction and observed slickenline = 2.7



Arrowsmith station 10
200
150
100
50
0
tau, MPa
0
50
100
150
200
250
300
sigma, MPa

Rheology:
“science of deformation and flow of matter”

or

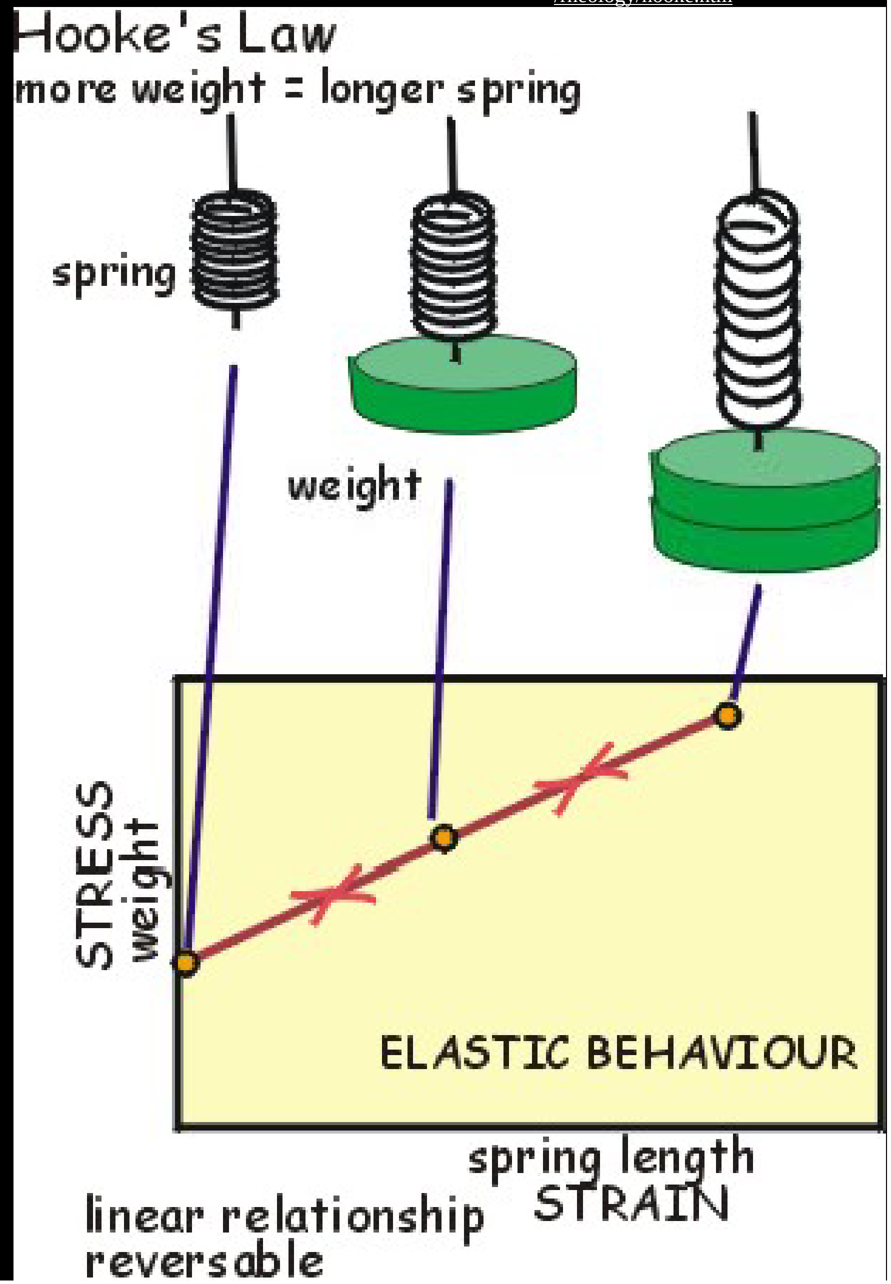
relationship between stress and strain

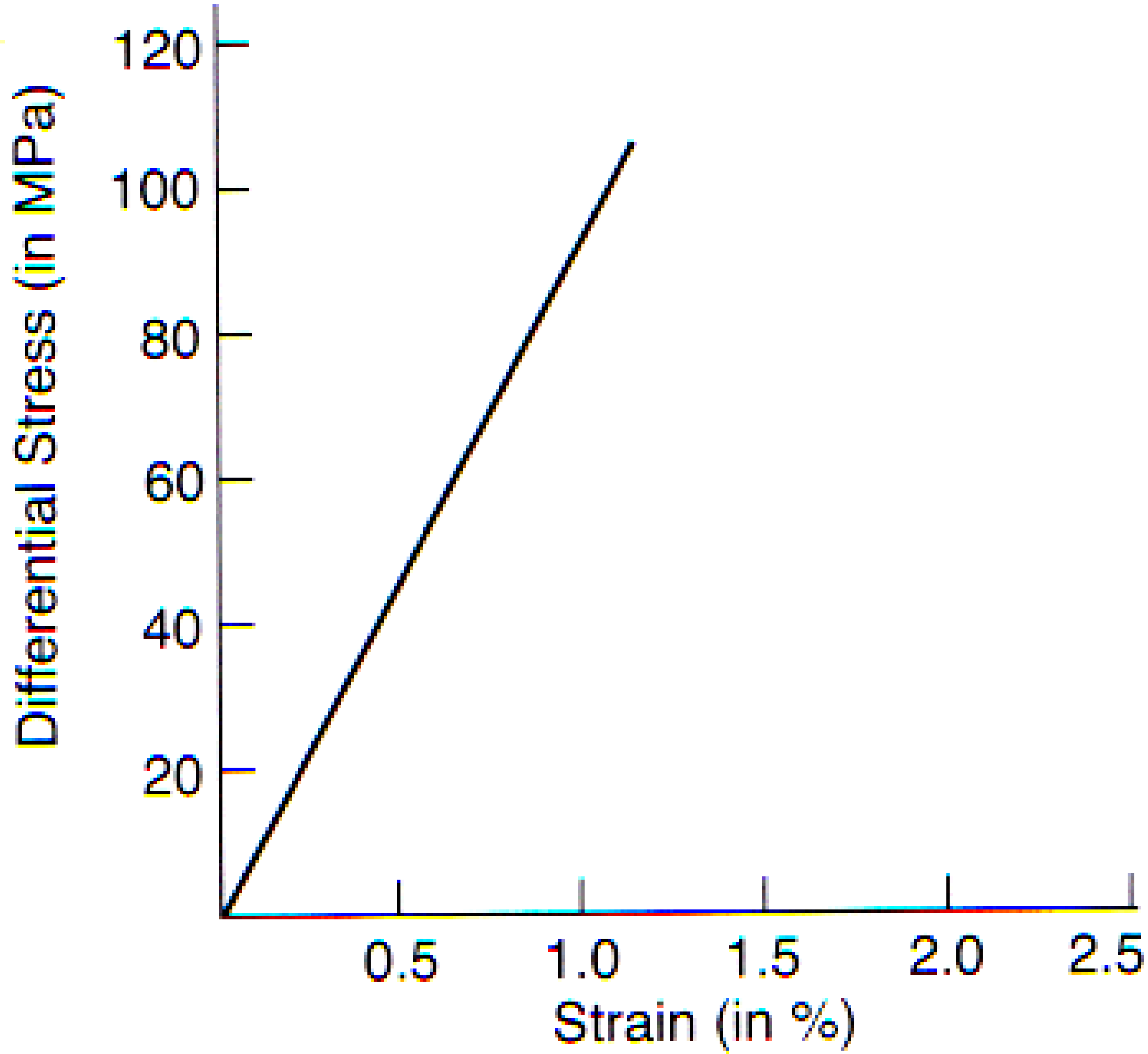
Idealized Elastic Material

http://www.see.leeds.ac.uk/structure/rheology/hooke.htm

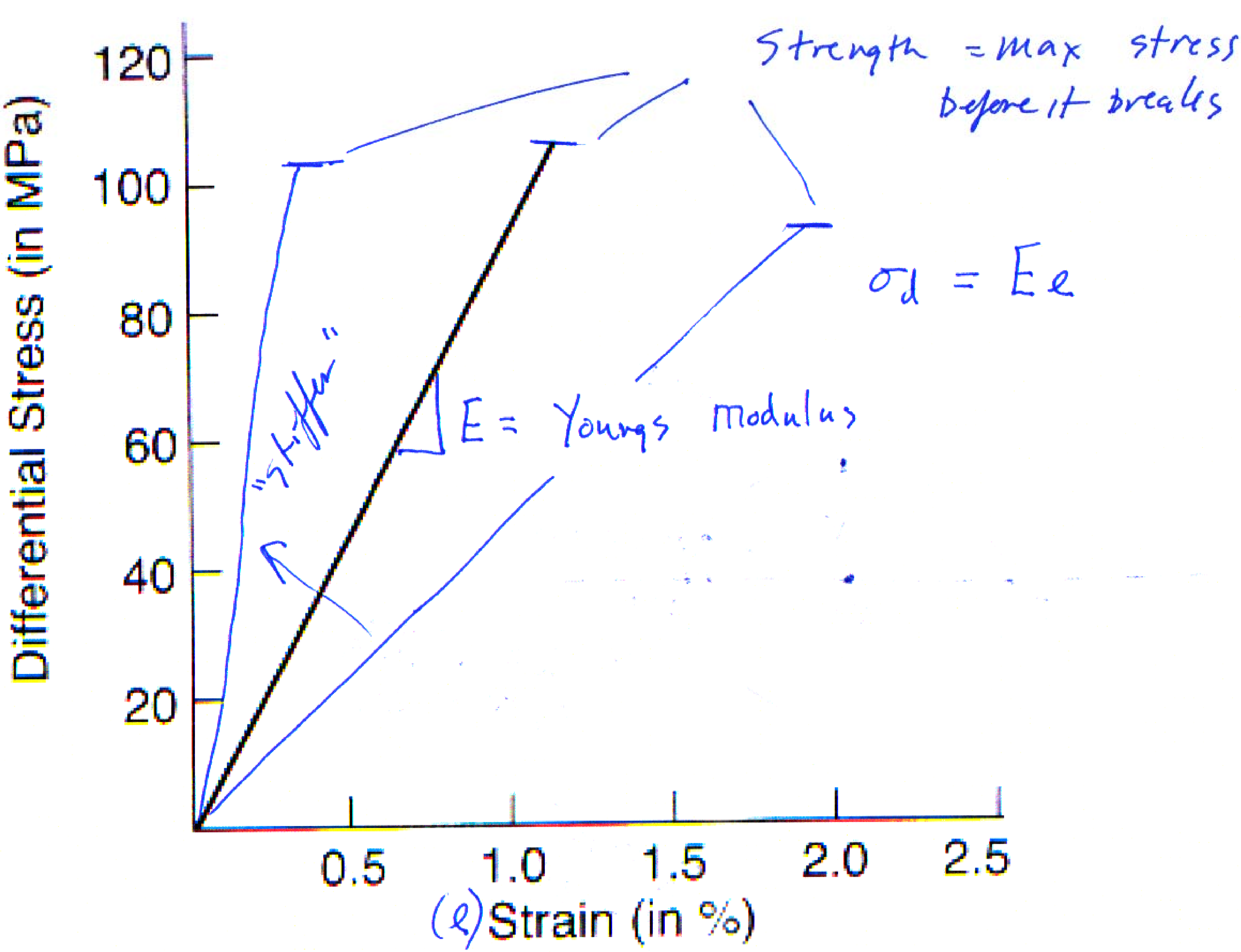
Linear relationship between force and extension (Hooke, 1676):

- Ut tensio sic uis
- **As extension so the force**
- Analogous to spring constant

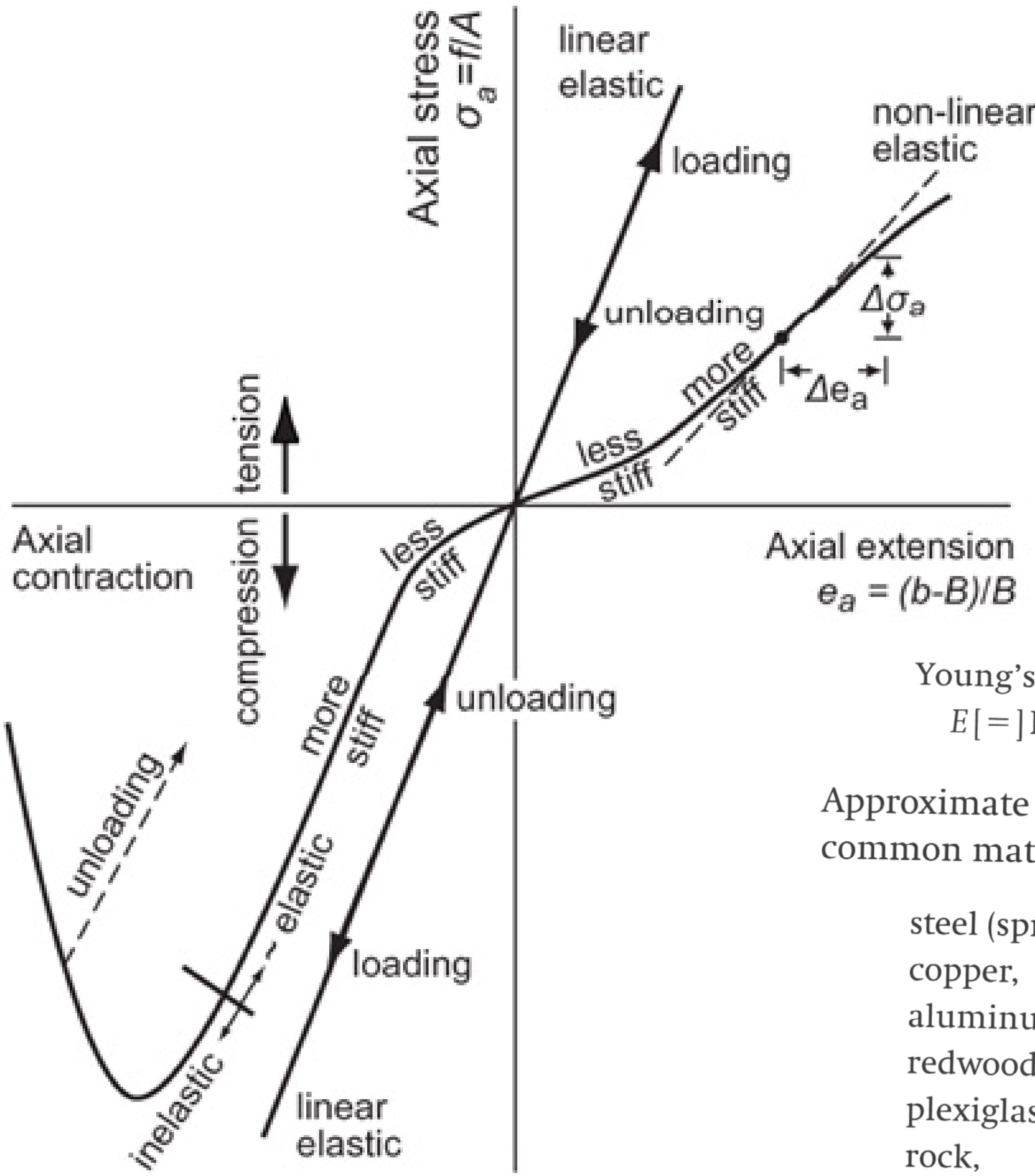




Stress/strain diagram.



Strength = max stress before it breaks
$\sigma_d = E e$
"stiffer"
E = Youngs modulus
Differential Stress (in MPa)
120
100
80
60
40
20
0.5
1.0
1.5
2.0
2.5
(e) Strain (in %)



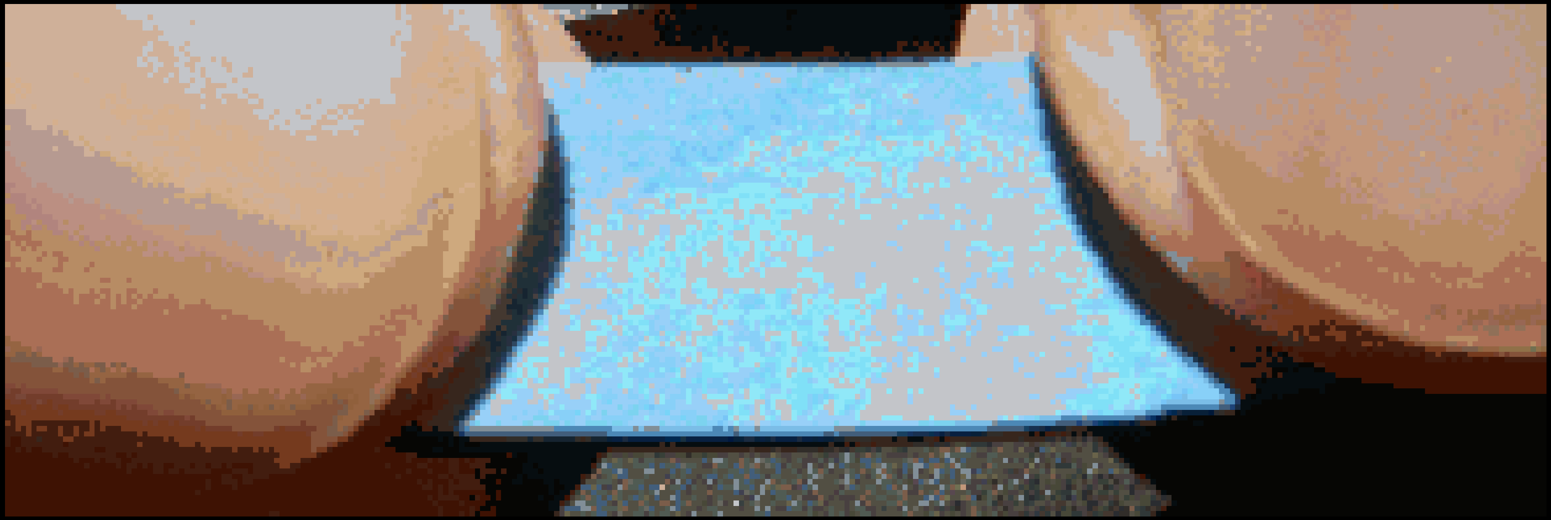
-Pollard and Fletcher, 2005

E (young's modulus)
= d σ_a/d e_a
Units of stress

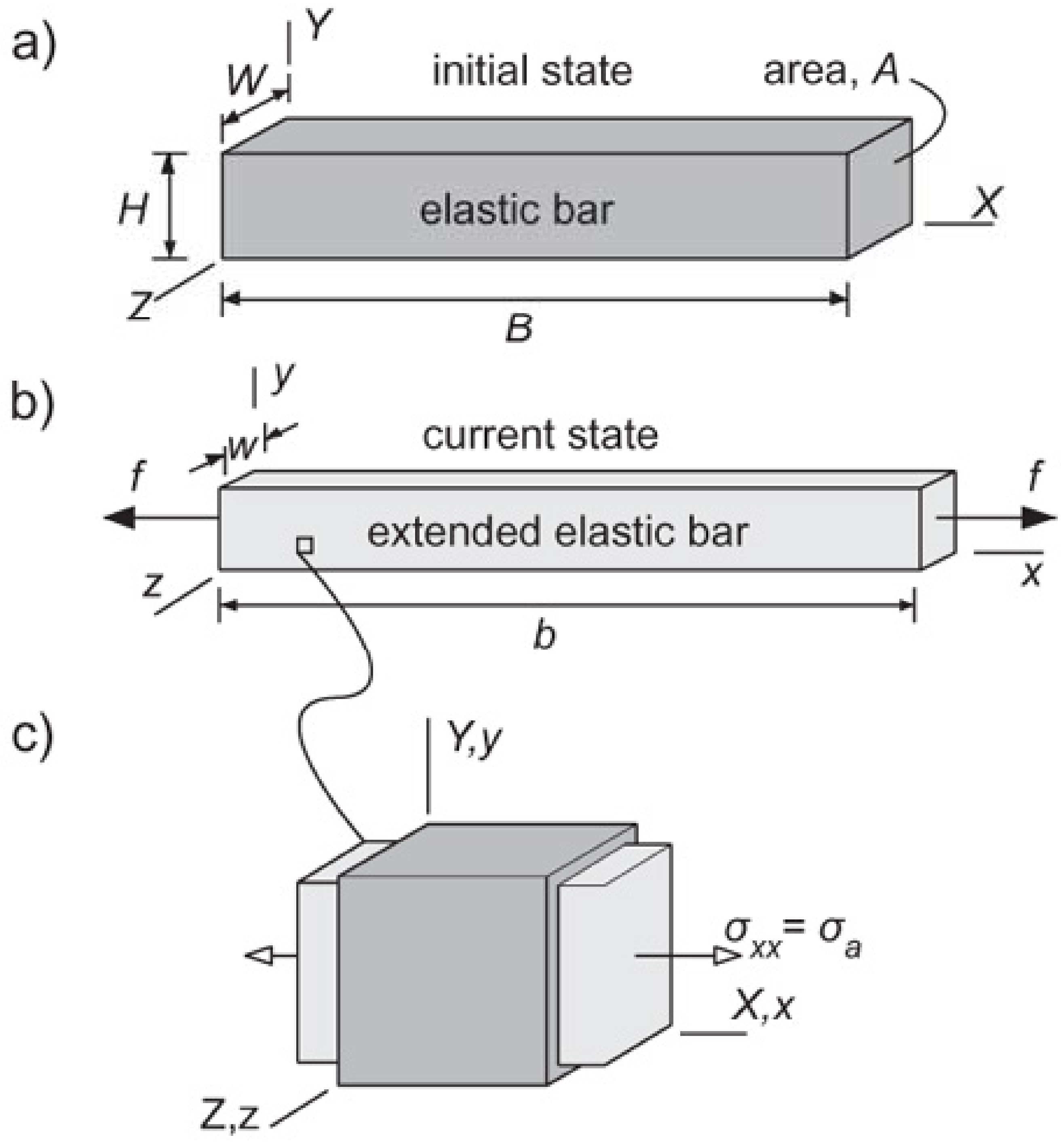
$$\text{Young's modulus, } E\,\{=\}\,\mathrm{ML^{-1}T^{-2}}, \quad \text{and} \quad E\,[=]\,\mathrm{Nm^{-2}} = \mathrm{Pa} \tag{8.8}$$

Approximate values of Young's modulus for common materials are (Eshbach, 1961):

steel (spring),	$E \sim 200$ GPa
copper,	$E \sim 110$ GPa
aluminum,	$E \sim 70$ GPa
redwood (dry),	$E \sim 9$ GPa
plexiglas,	$E \sim 3$ GPa
rock,	$E \sim 1$ to 100 GPa



http://silver.neep.wisc.edu/~lakes/PoissonIntro.html



Axial versus transverse deformation

σ_a = f/A

e_a = (b-B)/B

σ_p = 0

e_p = (w-W)/W

$\nu = - e_p / e_a$

(Poisson ratio)

Measures compressibility

rubber,	$\nu \sim 0.5$
cork,	$\nu \sim 0.0$
rock,	$\nu \sim 0.1$ to 0.3

Idealized Elastic Material

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right]$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu(\sigma_{zz} + \sigma_{xx})\right]$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})\right]$$

Stiffness and strength

"Lest there be any possible, probably, shadow of doubt, strength is not, repeat not, the same thing as stiffness. Stiffness, or Young's modulus or E, is concerned with how stiff, flexible, springy or floppy a material is. Strength is the force or stress needed to break a thing. A biscuit is stiff but weak, steel is stiff and strong, nylon is flexible (low E and strong), raspberry jelly is flexible (low E) and weak. The two properties together describe a solid about as well as you can reasonably expect two figures to do."

(Gordon, New science of strong materials, 2006)

Elastic constants

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \quad G = \frac{E}{2(1+\nu)}, \quad K = \frac{E}{3(1-2\nu)};$$

Lame's constant
Describes effects of dilatation on tensile stress

Shear modulus
Relates shear strain to shear stress

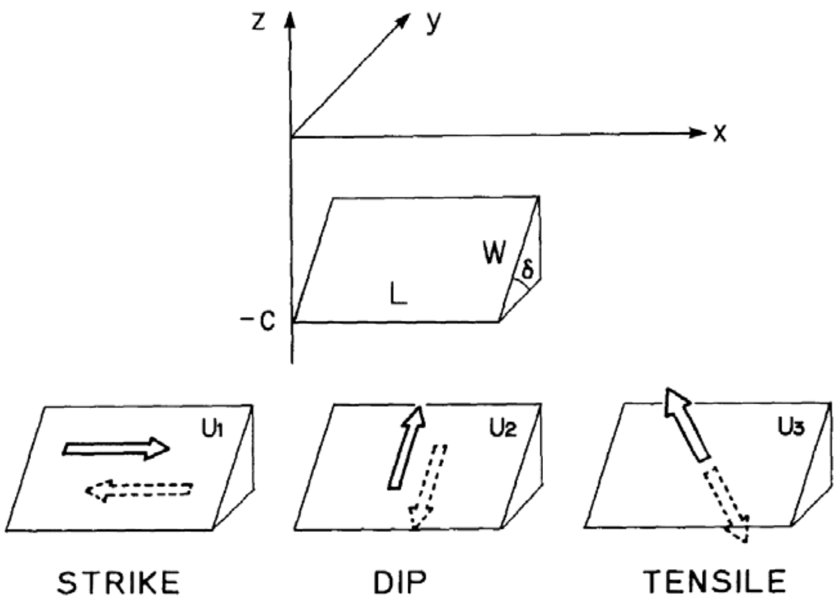
Bulk modulus
Relates volumetric strain to mean stress

E is Young's modulus which is the ratio of axial stress to axial strain
V is Poisson's ratio which is the negative of the ratio of transverse to longitudinal strain

--You only need two moduli to get the others

Okada, 1992 and 3D dislocations

- “Industry standard” for boundary element 3D elastic deformation modeling
- Linear elastic half space
- Rectangular elements
- Displacement boundary conditions
- Stress boundary conditions come from equivalent strain and displacement discontinuity



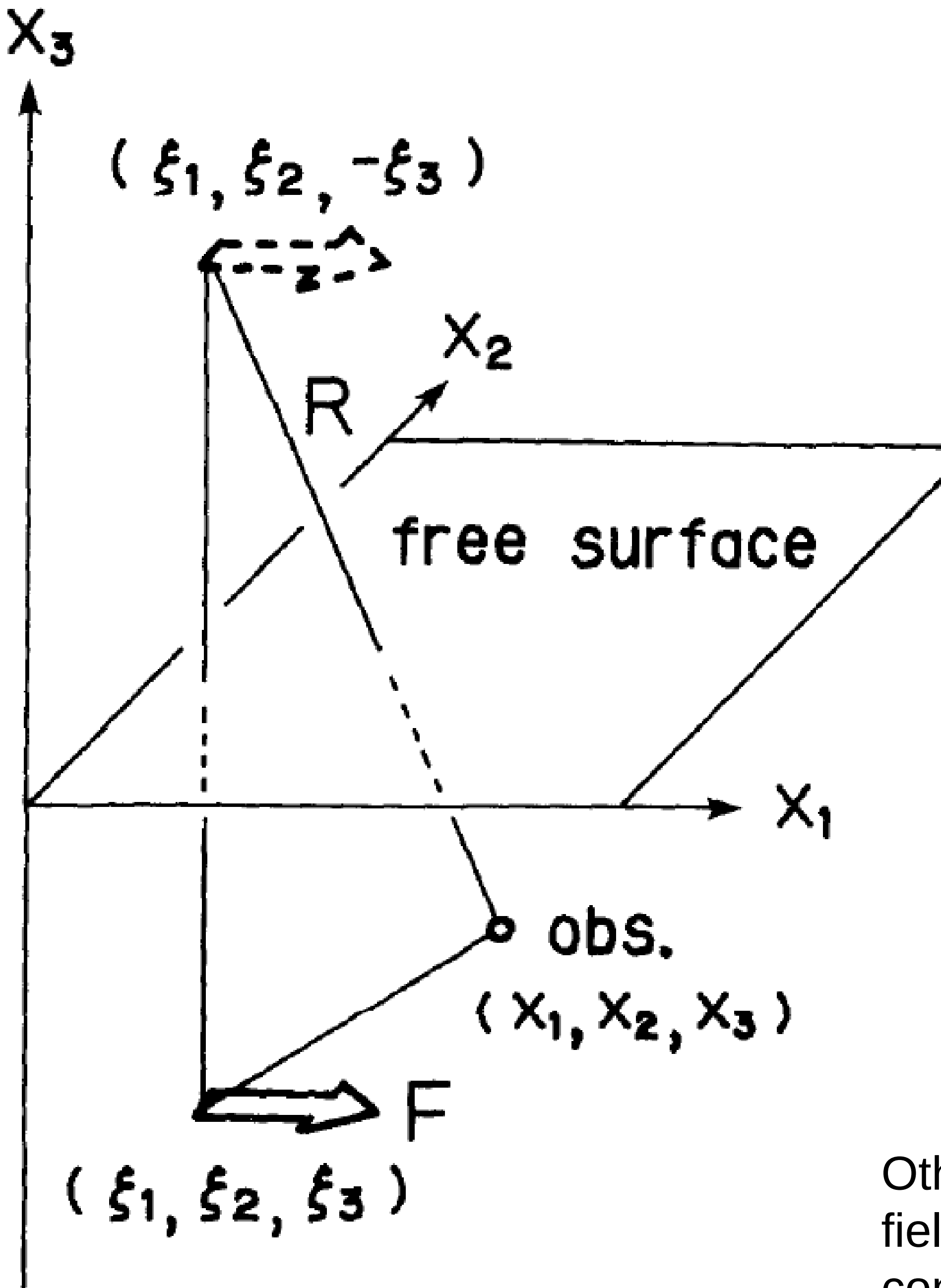


FIG. 1. A coordinate system adopted in this study.

$$
u_i^j(x_1, x_2, x_3) = u^j_{iA}(x_1, x_2, -x_3) - u^j_{iA}(x_1, x_2, x_3) + u^j_{iB}(x_1, x_2, x_3) + x_3 u^j_{iC}(x_1, x_2, x_3) \tag{1}
$$

$$
\begin{cases}
u^j_{iA} = \dfrac{F}{8\pi\mu}\left\{(2-\alpha)\dfrac{\delta_{ij}}{R} + \alpha\dfrac{R_i R_j}{R^3}\right\} \\
u^j_{iB} = \dfrac{F}{4\pi\mu}\left\{\dfrac{\delta_{ij}}{R} + \dfrac{R_i R_j}{R^3} + \dfrac{1-\alpha}{\alpha}\left[\dfrac{\delta_{ij}}{R+R_3} + \dfrac{R_i\delta_{j3} - R_j\delta_{i3}(1-\delta_{j3})}{R(R+R_3)} \right.\right. \\
\qquad \left.\left. - \dfrac{R_i R_j}{R(R+R_3)^2}(1-\delta_{i3})(1-\delta_{j3})\right]\right\} \\
u^j_{iC} = \dfrac{F}{4\pi\mu}(1-2\delta_{i3})\left\{(2-\alpha)\dfrac{R_i\delta_{j3} - R_j\delta_{i3}}{R^3} + \alpha\xi_3\left[\dfrac{\delta_{ij}}{R^3} - \dfrac{3R_i R_j}{R^5}\right]\right\},
\end{cases}
$$

where, $\alpha = (\lambda + \mu)/(\lambda + 2\mu)$; λ and μ are Lamé's constants; δ_{ij} is the Kronecker delta; and $R_1 = x_1 - \xi_1$, $R_2 = x_2 - \xi_2$, $R_3 = -x_3 - \xi_3$, $R^2 = R_1{}^2 + R_2{}^2 + R_3{}^2$.

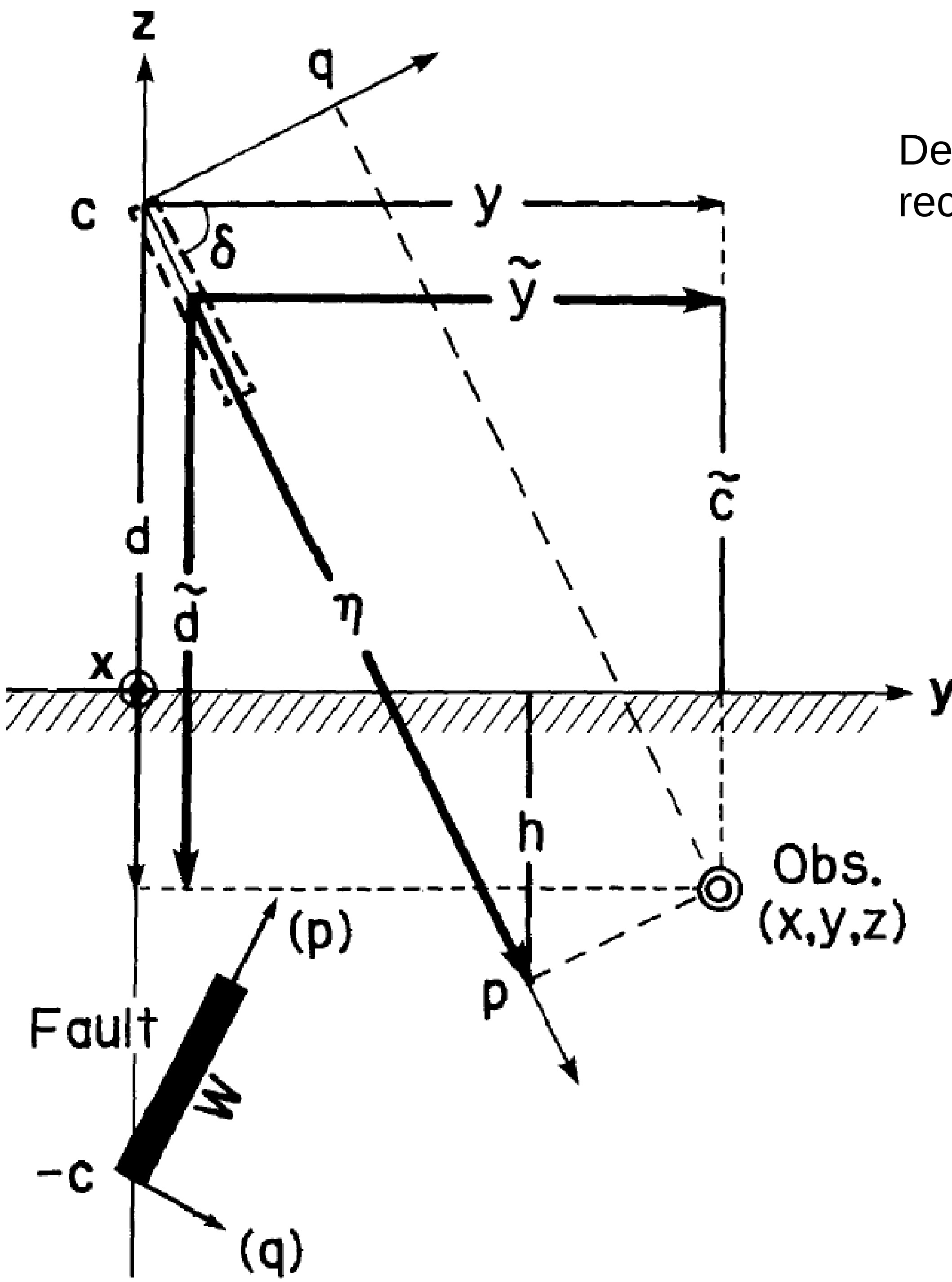
Displacement due to a point force
-ith component of displacement due to the point force in the jth direction

Part A: infinite medium terms
Part B: surface deformation related term
Part C: depth multiplied term

Other elastic field components

$$
e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right),
$$

$$
\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij}.
$$



Definition of geometry for rectangular source

Displacements due to discontinuity along finite rectangular source

--Integrate point sources along strike and dip

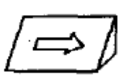
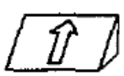
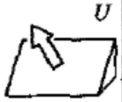
INTERNAL DISPLACEMENT FIELD DUE TO A FINITE RECTANGULAR SOURCE IN A HALF-SPACE. SEE TEXT AS TO THE MEANING OF THE TOP, MIDDLE, AND BOTTOM EQUATIONS IN EACH COMPARTMENT.

Displacement due to a Finite Fault at $(0,0,-c;\ \delta, L, W, U)$

$$\begin{cases} u_x(x,y,z) = U/2\pi\left[\, u_1^A - \hat{u}_1^A + u_1^B + z\,u_1^C \,\right] \\ u_y(x,y,z) = U/2\pi\left[\left(u_2^A - \hat{u}_2^A + u_2^B + z\,u_2^C\right)\cos\delta - \left(u_3^A - \hat{u}_3^A + u_3^B + z\,u_3^C\right)\sin\delta\right] \\ u_z(x,y,z) = U/2\pi\left[\left(u_2^A - \hat{u}_2^A + u_2^B - z\,u_2^C\right)\sin\delta + \left(u_3^A - \hat{u}_3^A + u_3^B - z\,u_3^C\right)\cos\delta\right] \end{cases}$$

$$d = c - z \qquad R^2 = \xi^2 + \eta^2 + q^2$$
$$p = y\cos\delta + d\sin\delta \qquad \tilde{y} = \eta\cos\delta + q\sin\delta$$
$$q = y\sin\delta - d\cos\delta \qquad \tilde{d} = \eta\sin\delta - q\cos\delta$$
$$\alpha = (\lambda+\mu)/(\lambda+2\mu) \qquad \tilde{c} = \tilde{d} + z$$

$$u_i^A = f_i^A(\xi,\eta,z)\Big|_{\xi=x}^{\xi=x-L}\Big|_{\eta=p}^{\eta=p-W} \qquad \hat{u}_i^A = f_i^A(\xi,\eta,-z)\Big\| \qquad u_i^B = f_i^B(\xi,\eta,z)\Big\| \qquad u_i^C = f_i^C(\xi,\eta,z)\Big\|$$

Type	f^A	f^B	f^C
Strike	$\frac{\Theta}{2} + \frac{\alpha}{2}\xi q Y_{11}$	$-\xi q Y_{11} - \Theta \quad -\frac{1-\alpha}{\alpha} I_1 \sin\delta$	$(1-\alpha)\,\xi Y_{11}\cos\delta \quad -\alpha\,\xi q Z_{32}$
U	$\frac{\alpha}{2}\frac{q}{R}$	$-\frac{q}{R} \quad +\frac{1-\alpha}{\alpha}\frac{\tilde{y}}{R+\tilde{d}}\sin\delta$	$(1-\alpha)\left[\frac{\cos\delta}{R} + 2qY_{11}\sin\delta\right] - \alpha\frac{\tilde{c}q}{R^3}$
	$\frac{1-\alpha}{2}\ln(R+\eta) - \frac{\alpha}{2}q^2 Y_{11}$	$q^2 Y_{11} \quad -\frac{1-\alpha}{\alpha} I_2 \sin\delta$	$(1-\alpha)\,qY_{11}\cos\delta \quad -\alpha\left[\frac{\tilde{c}\eta}{R^3} - zY_{11} + \xi^2 Z_{32}\right]$
Dip	$\frac{\alpha}{2}\frac{q}{R}$	$-\frac{q}{R} \quad +\frac{1-\alpha}{\alpha} I_3 \sin\delta\cos\delta$	$(1-\alpha)\frac{\cos\delta}{R} \quad - qY_{11}\sin\delta - \alpha\frac{\tilde{c}q}{R^3}$
U	$\frac{\Theta}{2} + \frac{\alpha}{2}\eta q X_{11}$	$-\eta q X_{11} - \Theta \quad -\frac{1-\alpha}{\alpha}\frac{\xi}{R+\tilde{d}}\sin\delta\cos\delta$	$(1-\alpha)\,\tilde{y}X_{11} \quad -\alpha\,\tilde{c}\eta q X_{32}$
	$\frac{1-\alpha}{2}\ln(R+\xi) - \frac{\alpha}{2}q^2 X_{11}$	$q^2 X_{11} \quad +\frac{1-\alpha}{\alpha} I_4 \sin\delta\cos\delta$	$-\tilde{d}X_{11} - \xi Y_{11}\sin\delta - \alpha\,\tilde{c}\left[X_{11} - q^2 X_{32}\right]$
Tensile	$-\frac{1-\alpha}{2}\ln(R+\eta) - \frac{\alpha}{2}q^2 Y_{11}$	$q^2 Y_{11} \quad -\frac{1-\alpha}{\alpha} I_3 \sin^2\delta$	$-(1-\alpha)\left[\frac{\sin\delta}{R} + qY_{11}\cos\delta\right] - \alpha\left[zY_{11} - q^2 Z_{32}\right]$
U	$-\frac{1-\alpha}{2}\ln(R+\xi) - \frac{\alpha}{2}q^2 X_{11}$	$q^2 X_{11} \quad +\frac{1-\alpha}{\alpha}\frac{\xi}{R+\tilde{d}}\sin^2\delta$	$(1-\alpha)\,2\xi Y_{11}\sin\delta + \tilde{d}X_{11} - \alpha\,\tilde{c}\left[X_{11} - q^2 X_{32}\right]$
	$\frac{\Theta}{2} - \frac{\alpha}{2}q(\eta X_{11} + \xi Y_{11})$	$q(\eta X_{11} + \xi Y_{11}) - \Theta - \frac{1-\alpha}{\alpha} I_4 \sin^2\delta$	$(1-\alpha)\left[\tilde{y}X_{11} + \xi Y_{11}\cos\delta\right] + \alpha\, q\left[\tilde{c}\eta X_{32} + \xi Z_{32}\right]$

$$\Theta = tan^{-1}\frac{\xi\eta}{qR} \qquad I_1 = -\frac{\xi}{R+\tilde{d}}\cos\delta - I_4\sin\delta \qquad I_2 = \ln(R+\tilde{d}) + I_3\sin\delta$$

$$I_3 = \frac{1}{\cos\delta}\frac{\tilde{y}}{R+\tilde{d}} - \frac{1}{\cos^2\delta}\left[\ln(R+\eta) - \sin\delta\ln(R+\tilde{d})\right] \qquad \left(I_3 = \frac{1}{2}\left[\frac{\eta}{R+\tilde{d}} + \frac{\tilde{y}q}{(R+\tilde{d})^2} - \ln(R+\eta)\right] \quad \text{if } \cos\delta = 0\right)$$

$$X^2 = \xi^2 + q^2 \qquad I_4 = \frac{\sin\delta}{\cos\delta}\frac{\xi}{R+\tilde{d}} + \frac{2}{\cos^2\delta}\tan^{-1}\frac{\eta(X+q\cos\delta) + X(R+X)\sin\delta}{\xi(R+X)\cos\delta} \qquad \left(I_4 = \frac{1}{2}\frac{\xi\tilde{y}}{(R+\tilde{d})^2} \quad \text{if } \cos\delta = 0\right)$$

Strike direction

Dip direction (C: image)

Opening direction (C: image)

Part A: infinite medium terms

Part B: surface deformation related term

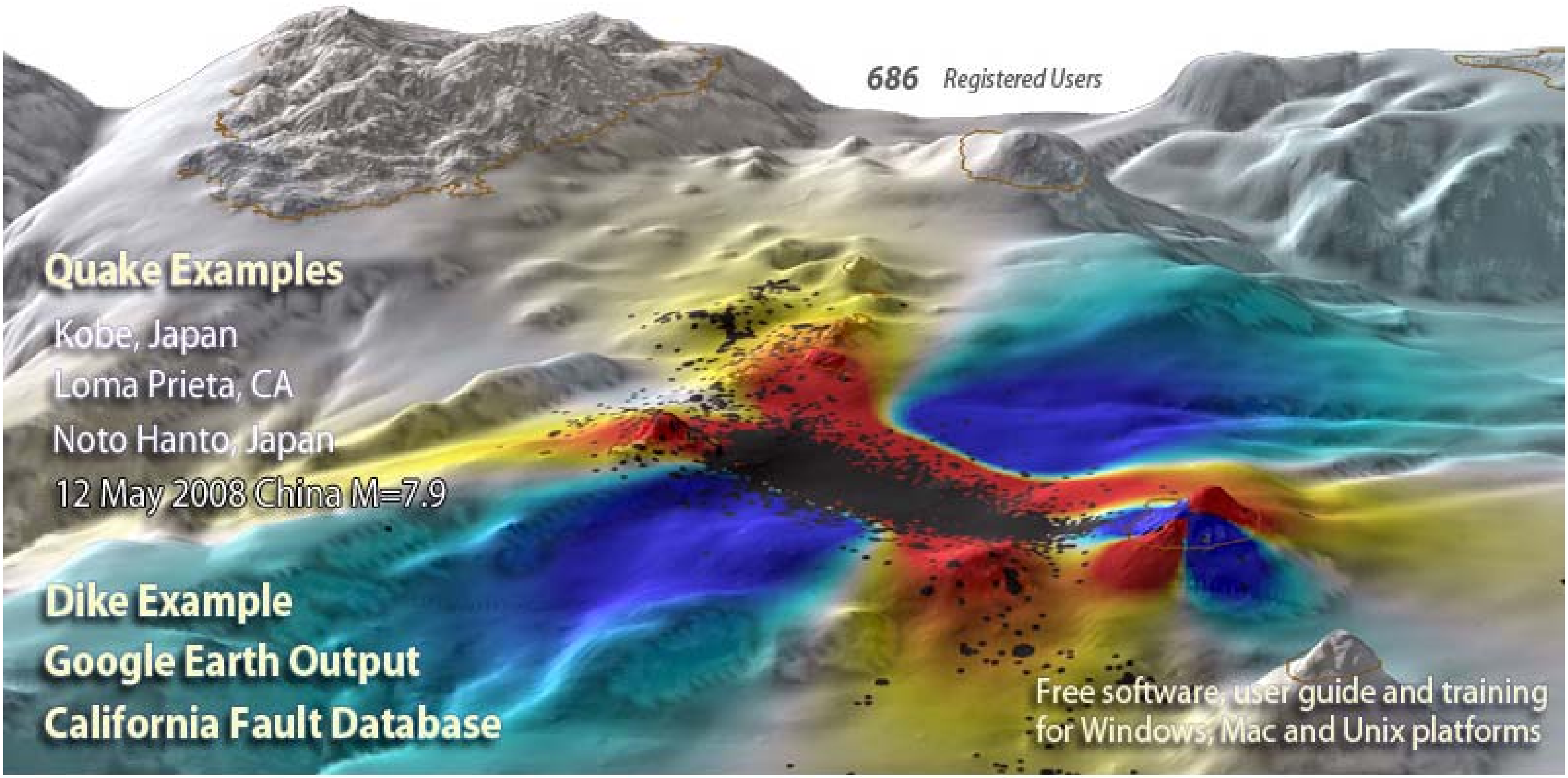
Part C: depth multiplied term

Coulomb 3

Graphic-rich deformation & stress-change software
for earthquake, tectonic, and volcano research & teaching

Shinji Toda, Ross Stein, Jian Lin, and Volkan Sevilgen
AIST USGS WHOI USGS

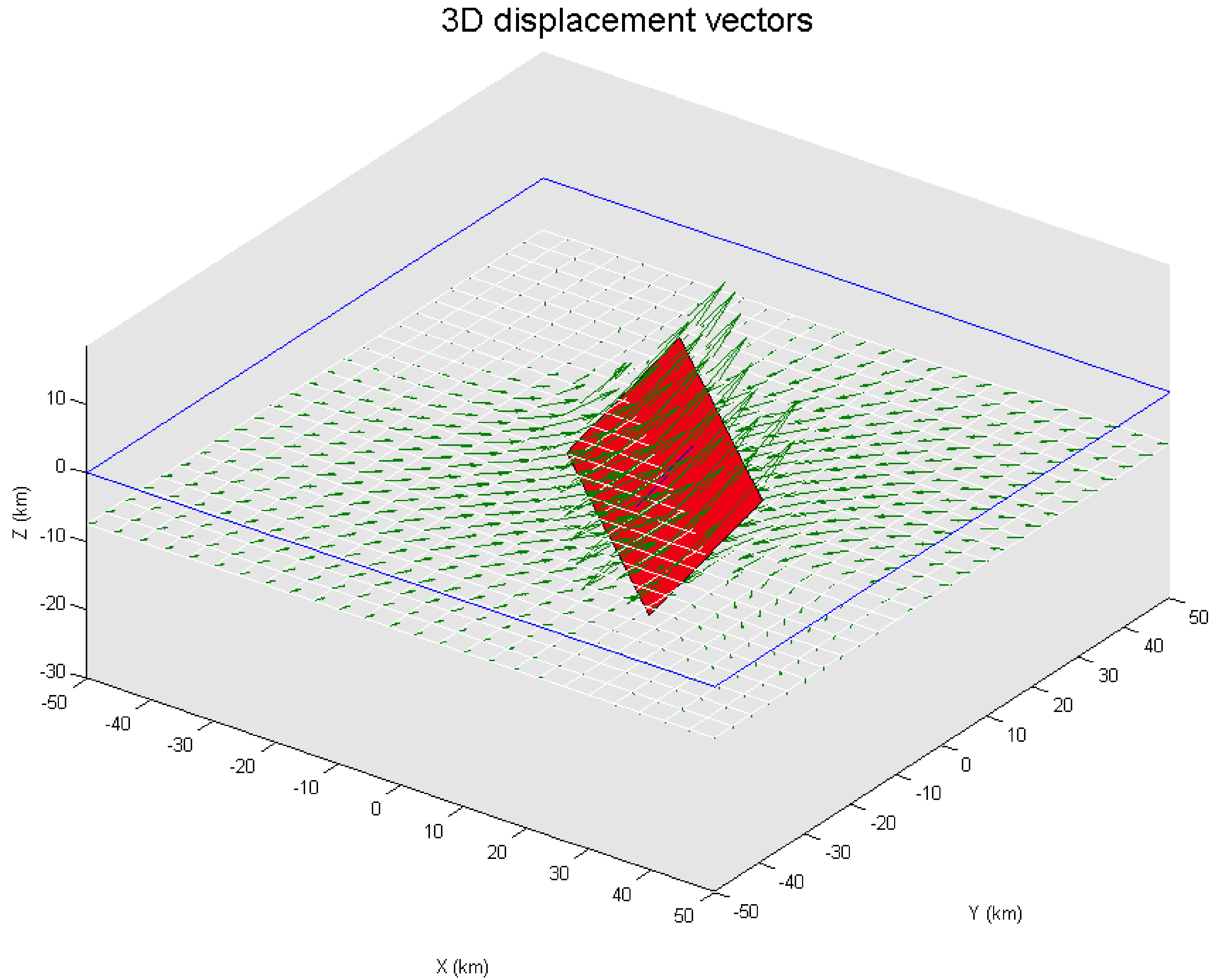
Overview **What's new?** **Scientific Background** **Download & Support** **Training**



Funding by the
U.S. Office of Foreign Disaster Assistance
is gratefully acknowledged

http://quake.usgs.gov/research/deformation/modeling/coulomb/index.html

Coulomb


3D displacement vectors
Z (km)
10
0
-10
-20
-30
X (km)
-50
-40
-30
-20
-10
0
10
20
30
40
50
Y (km)
-50
-40
-30
-20
-10
0
10
20
30
40
50

Softwares (Poly3D, Poly3Dinv & Dynel) - Mozilla Firefox

File Edit View Go Bookmarks Tools Help

http://pangea.stanford.edu/research/geomech/Software/Software.htm

Stanford University | Home | Department | Earth Sci. | Stanford

Earth Sciences

Structural Geology and Geomechanics

Software

- Program Description
- Who we are
- Rock Fracture Project
- Research

Upcoming Events

Poly3D

- Introduction
- Poly3DGUI
- Examples
  - Chinnery's model
  - 3D normal fault
- Applications
  - Fault interaction and slip
  - 1999 Turkish earthquake
  - Fault linkage in reservoirs
  - Subseismic fault modeling

Poly3Dinv

- Introduction
- InSAR addon
- Examples
  - Hector Mine earthquake
  - Chi-Chi earthquake
  - Kozani earthquake

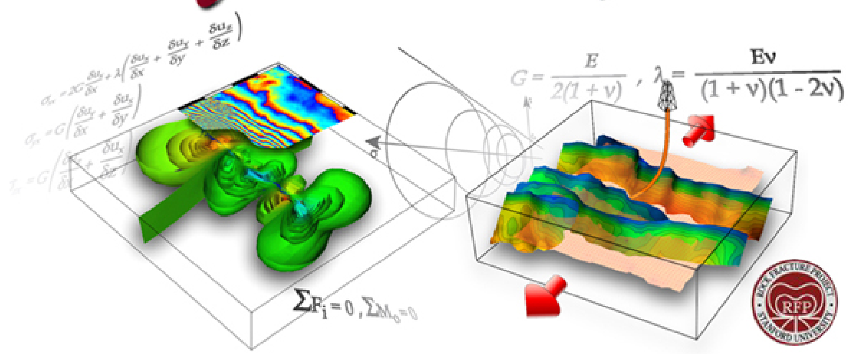
Poly3DpaleoStress

Poly3Dfriction

Dynel

- Introduction
- DynelGUI
- Examples
  - 2D/3D restoration
  - Quartz grain crushing
  - Forward modeling
  - Present day stress
- Applications
  - Stick slip dodeling

Poly3D and Poly3DGUI



Great Research Tool for Fracture Mechanics and Earthquake Studies

and

Industry's Choice for Geomechanical Analysis

of Fractured Oil and Gas Reservoirs

(last update: 08-20-2002)

Done

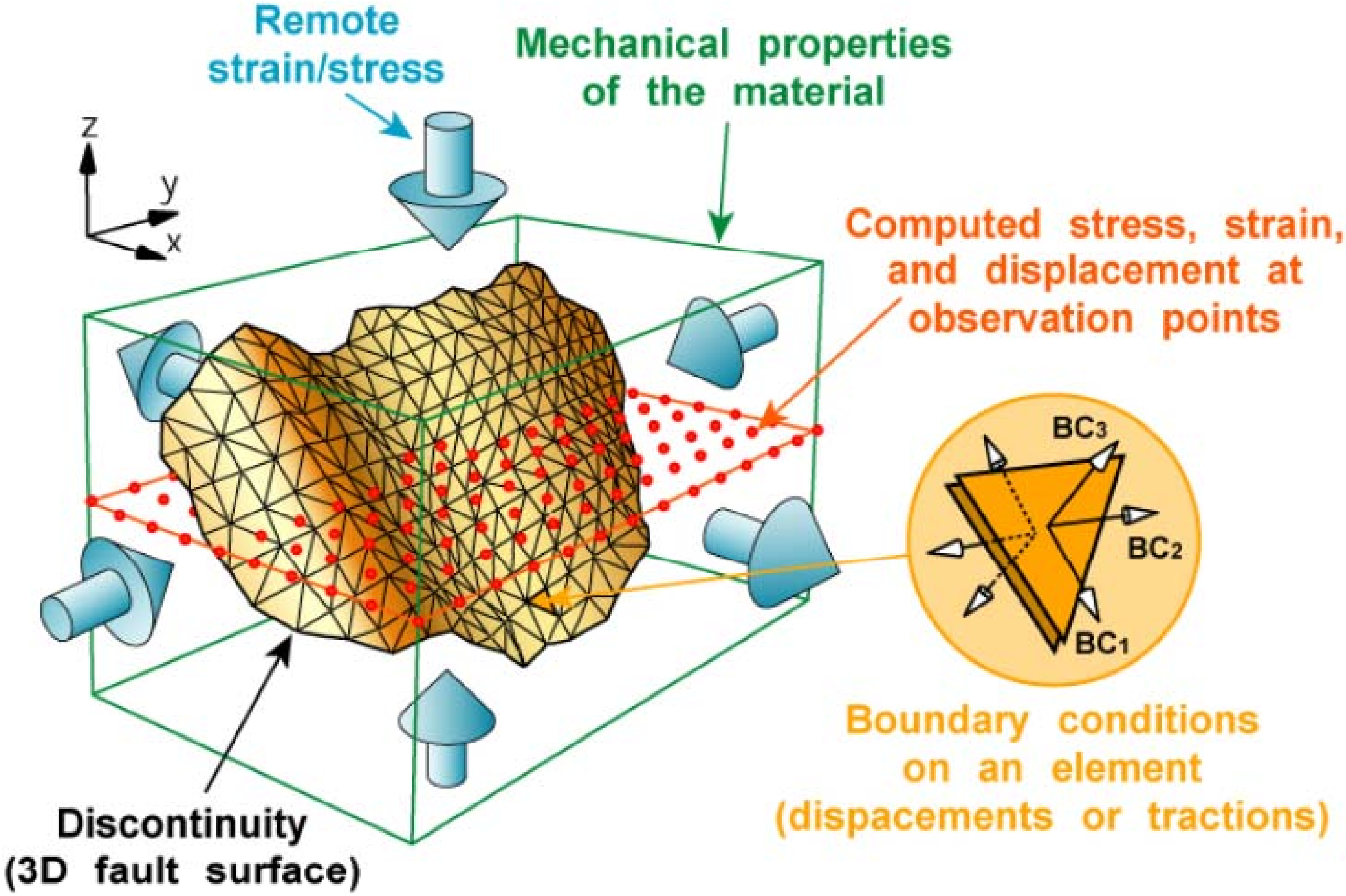
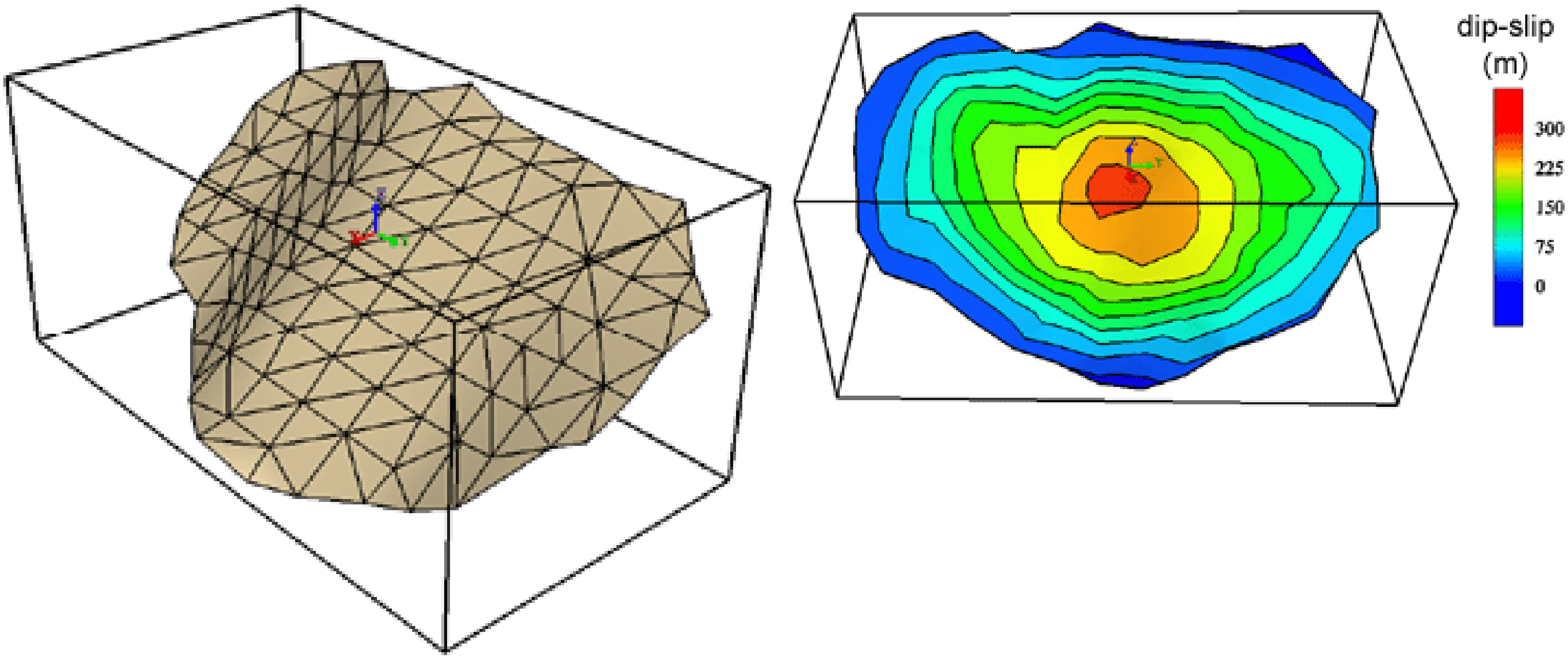
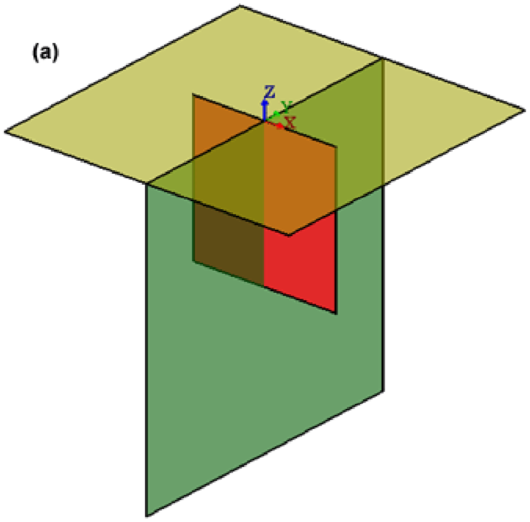


Figure 1: Poly3D model configuration for a 3D fault.

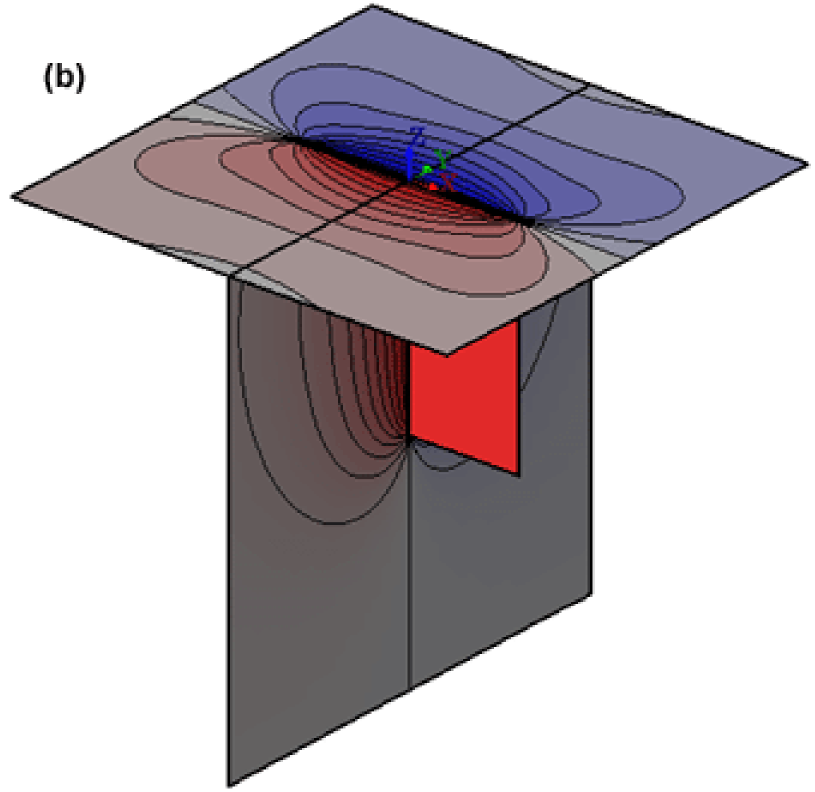


3D normal fault with constant stress drop

(a)

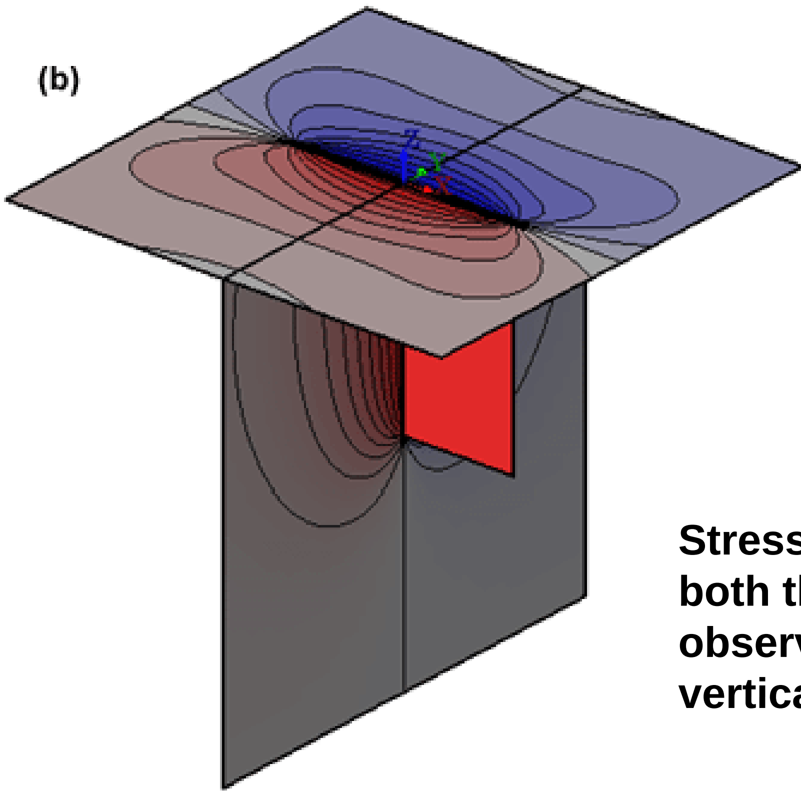


(b)

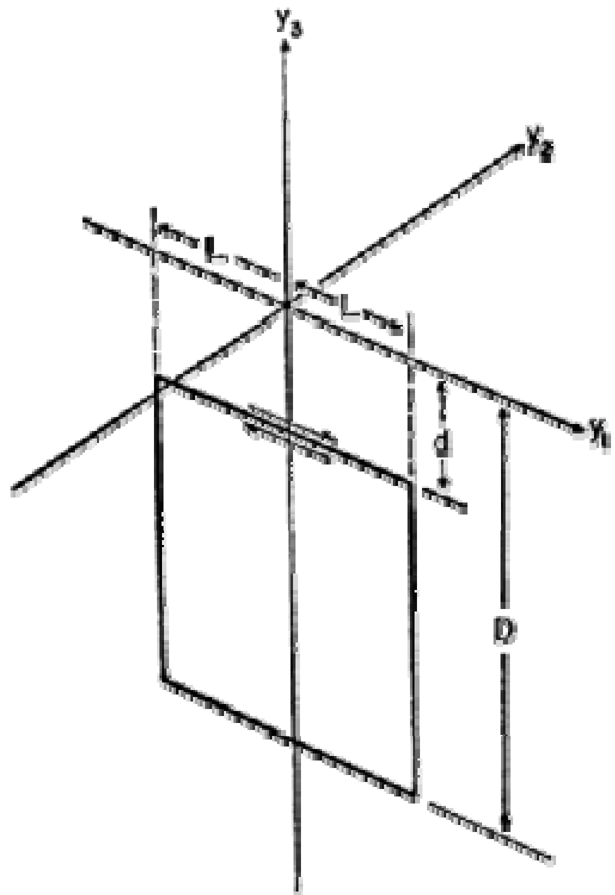


Poly3D color figure of the displacement Ux at both the free surface observation plane and the vertical observation plane

(b)



Stress component Sxy at both the free surface observation plane and the vertical observation plane



Chinnery’s fault

Bulletin of the Seismological Society of America, Vol. 84, No. 3, pp. 935–953, June 1994

Static Stress Changes and the Triggering of Earthquakes

by Geoffrey C. P. King, Ross S. Stein, and Jian Lin

Key concepts:
- •Source faults
- •Receiver faults
- •Optimally oriented faults
- •Assume receiver faults are close to failure
- •Triggering lag time is a problem

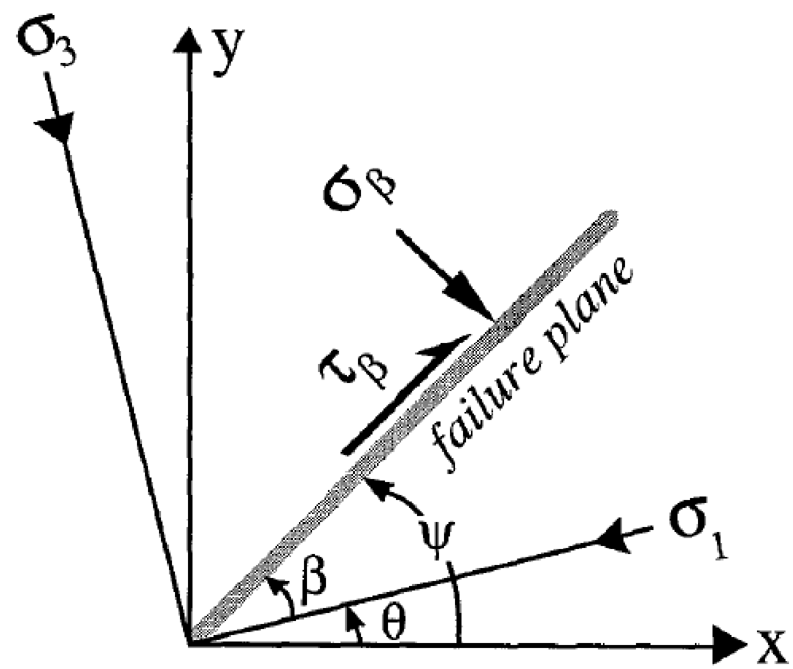


Figure 1. The axis system used for calculations of Coulomb stresses on optimum failure planes. Compression and right-lateral shear stress on the failure plane are taken as positive. The sign of τ_β is reversed for calculations of right-lateral Coulomb failure on specified failure planes.

Change of coulomb stress on faults of specified orientation

Given $\mathbf{S}$ principal stress tensor with orientation $x'y'z'$

Rotate to N-S, E-W components

Can change spatially
Remote:

Induced:

Total:

$$\mathbf{R} = \begin{bmatrix} l & l' & l'' \\ m & m' & m'' \\ n & n' & n'' \end{bmatrix} \text{where} \quad \begin{aligned} l &= x * x' \\ l' &= x * y' \\ l'' &= x * z' \\ \ldots \end{aligned}$$

$$\mathbf{S}' = \mathbf{R}^T\mathbf{S}\mathbf{R}$$

And given plane with normal vector direction cosines $\mathbf{N}$

Can change spatially

Traction $\mathbf{T} = \mathbf{S}' * \mathbf{N}$ (row and column multiplication)
$T = \sqrt{\mathbf{T}(1)^2 + \mathbf{T}(2)^2 + \mathbf{T}(3)^2}$ traction magnitude

$\sigma_n = \mathbf{T} \cdot \mathbf{N}$ dot product for normal traction magnitude

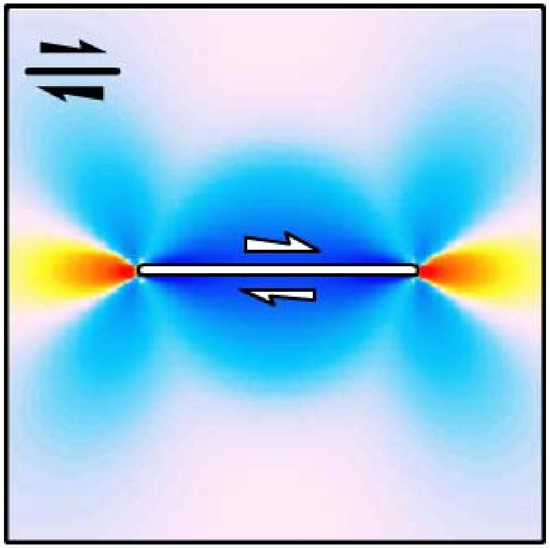
$\mathbf{B} = \mathbf{T} \times \mathbf{N}$ cross product for null vector
$B = \sqrt{\mathbf{B}(1)^2 + \mathbf{B}(2)^2 + \mathbf{B}(3)^2}$ $\mathbf{B}$ magnitude
$\mathbf{B}_{normalized} = \mathbf{B}./B$ normalize for orientation if necessary

$\mathbf{T_s} = \mathbf{N} \times \mathbf{B}$ cross product for shear traction vector
$\tau = \sqrt{\mathbf{T_s}(1)^2 + \mathbf{T_s}(2)^2 + \mathbf{T_s}(3)^2}$ shear traction magnitude
$\mathbf{T}_{snormalized} = \mathbf{T_s}./\tau$ normalize for shear traction orientation

Coulomb failure function: $\Delta\sigma_f = \Delta\tau - (\mu - P)\Delta\sigma_n$

How the Coulomb Stress Change is Calculated

Stress Rise Drop



Shear stress change

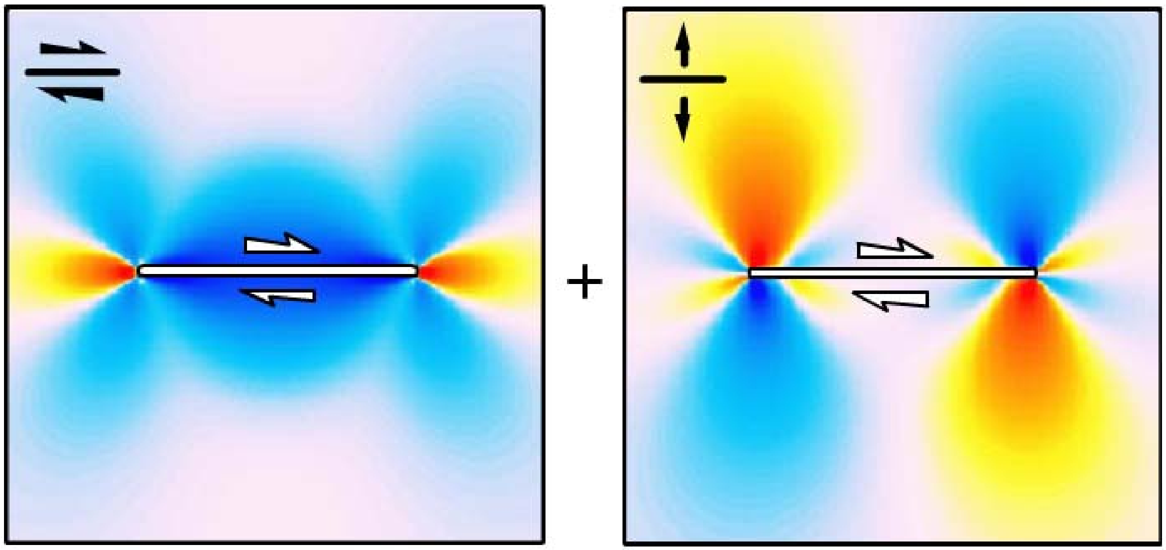
$\Delta\tau_s$

- Example calculation for faults parallel to master fault

From King et al (BSSA, 1994)

How the Coulomb Stress Change is Calculated

Stress Rise Drop



Shear stress change + Friction coefficient **x** normal stress change

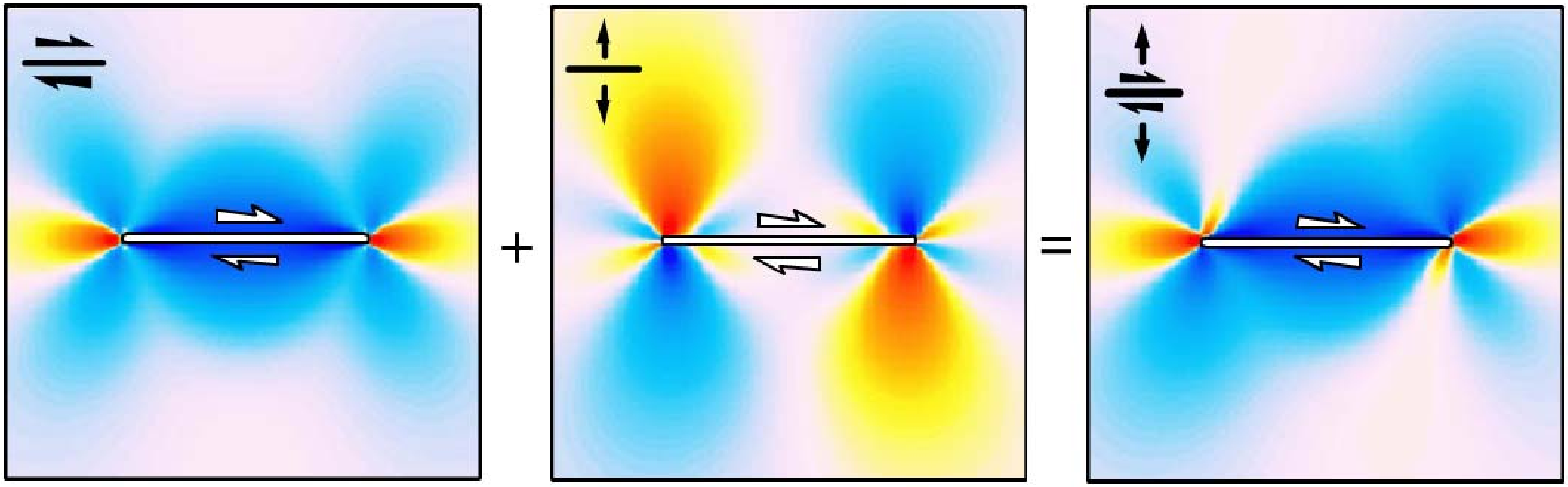
$$\Delta\tau_s \quad + \quad \mu' \, (\Delta\sigma_n)$$

- Example calculation for faults parallel to master fault

From King et al (BSSA, 1994)

How the Coulomb Stress Change is Calculated

Stress Rise Drop



Shear stress change + Friction coefficient **x** normal stress change = Coulomb failure stress change

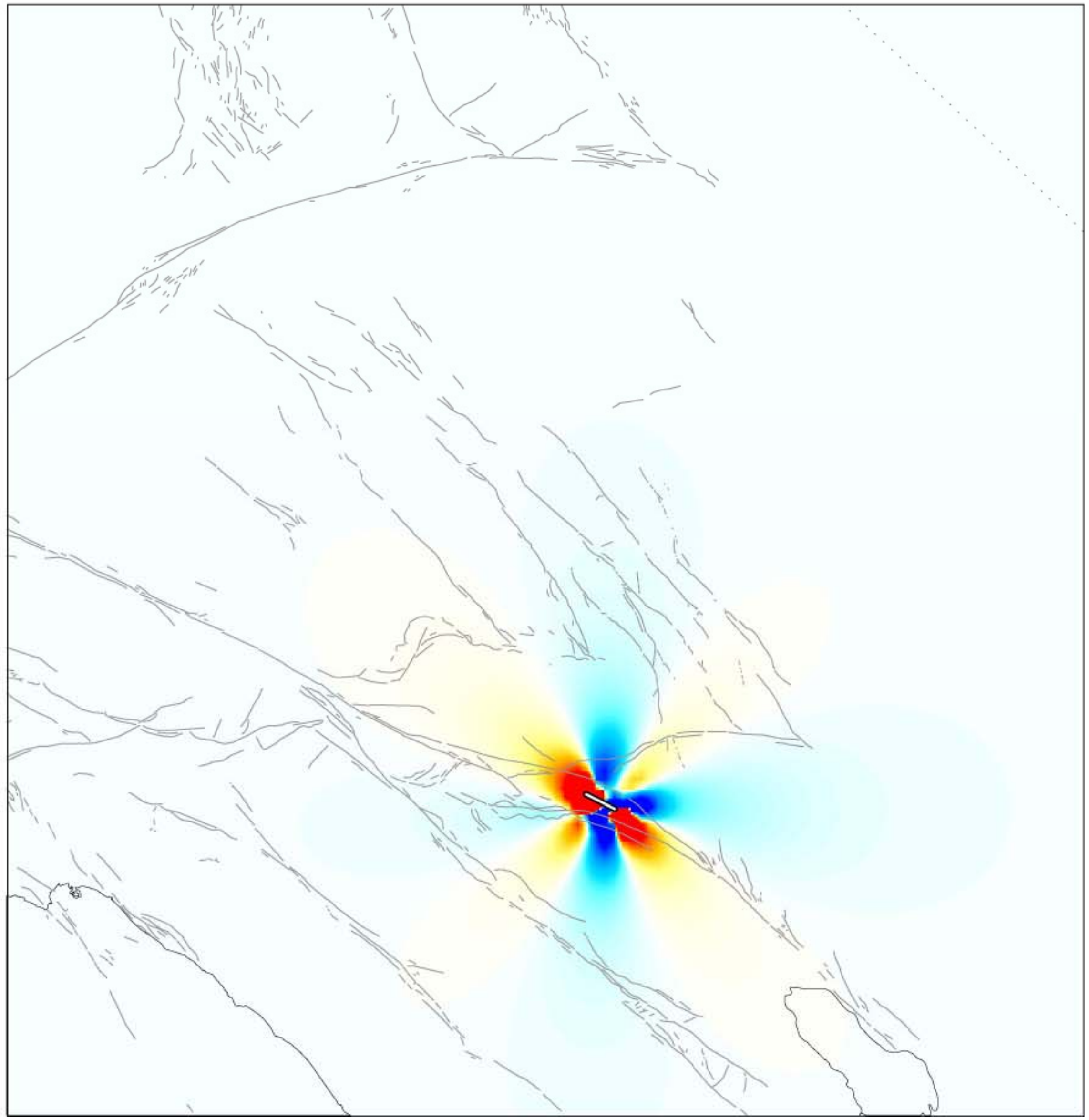
$$\Delta\tau_s + \mu' (\Delta\sigma_n) = \Delta\sigma_f$$

- Example calculation for faults parallel to master fault

From King et al (BSSA, 1994)

Coulomb stress imparted by mainshocks

1986 **M**=6.0 North Palm Springs



Source fault

Distance (km)

0 50 100

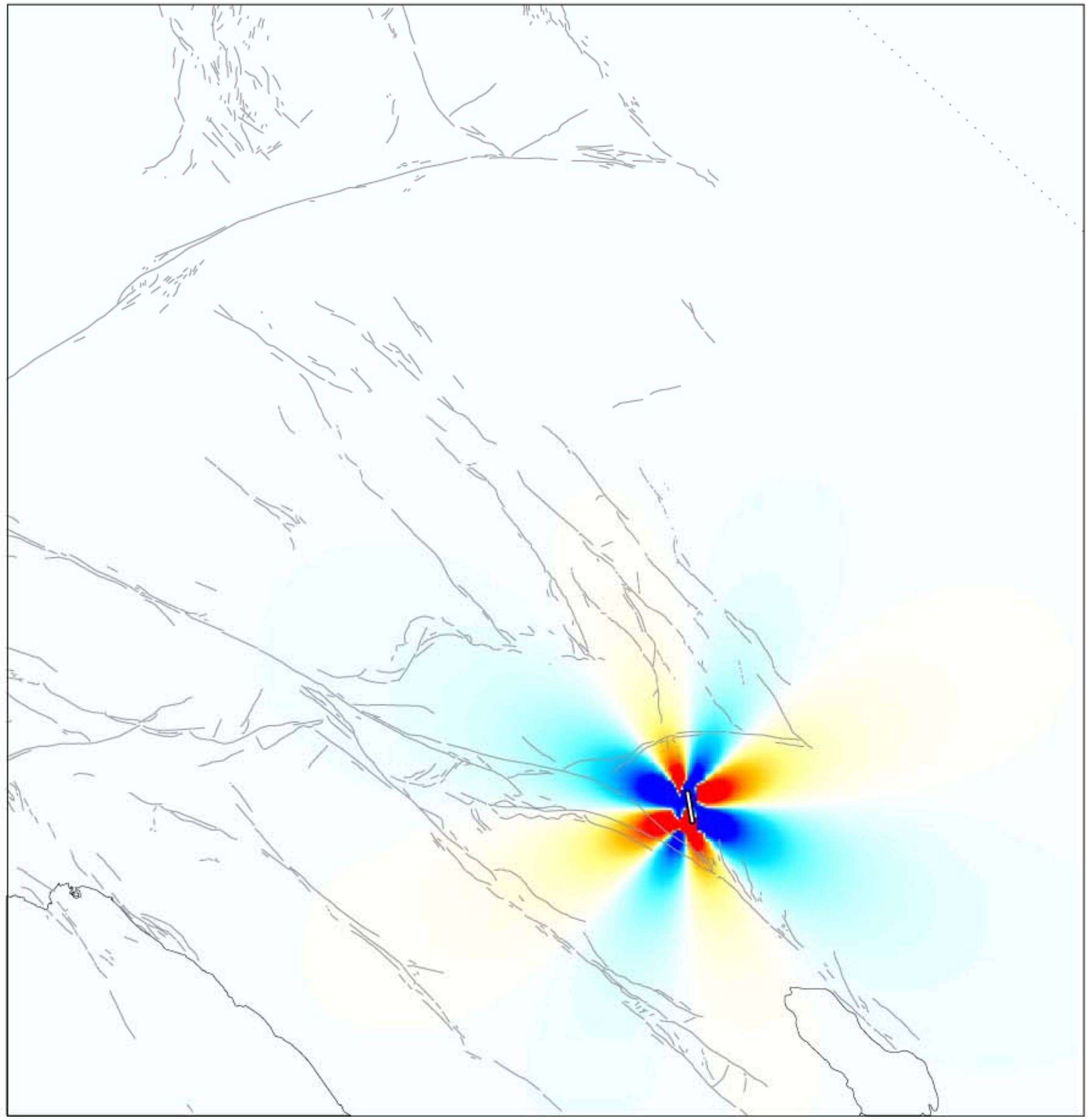
Coulomb stress change (bars)

-1.0 -0.5 0.0 0.5 1.0

from Todal et al (JGR, 2005)

Coulomb stress imparted by mainshocks

1992 **M**=6.2 Joshua Tree

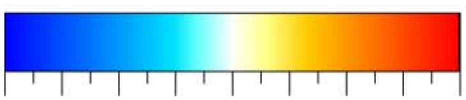


Source fault

Distance (km)

0 50 100

Coulomb stress change (bars)



-1.0 -0.5 0.0 0.5 1.0

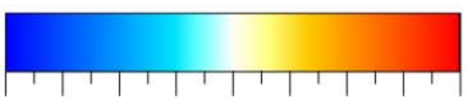
from Todal et al (JGR, 2005)

Coulomb stress imparted by mainshocks

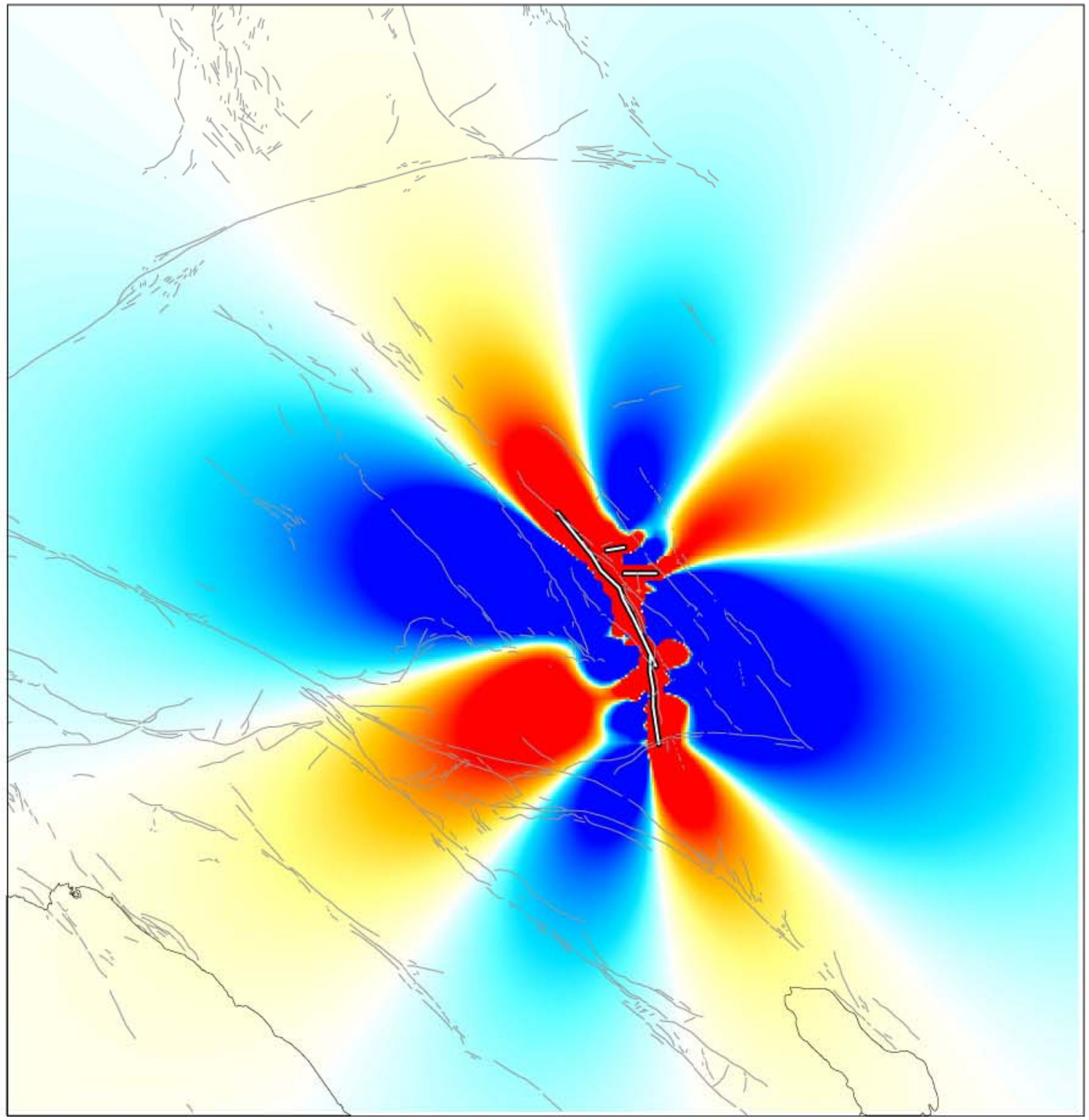
Source fault

Distance (km)

Coulomb stress change (bars)

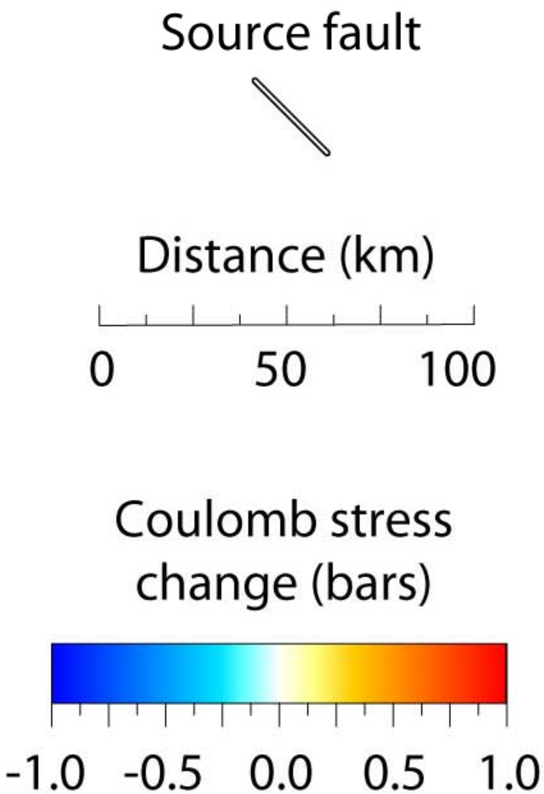


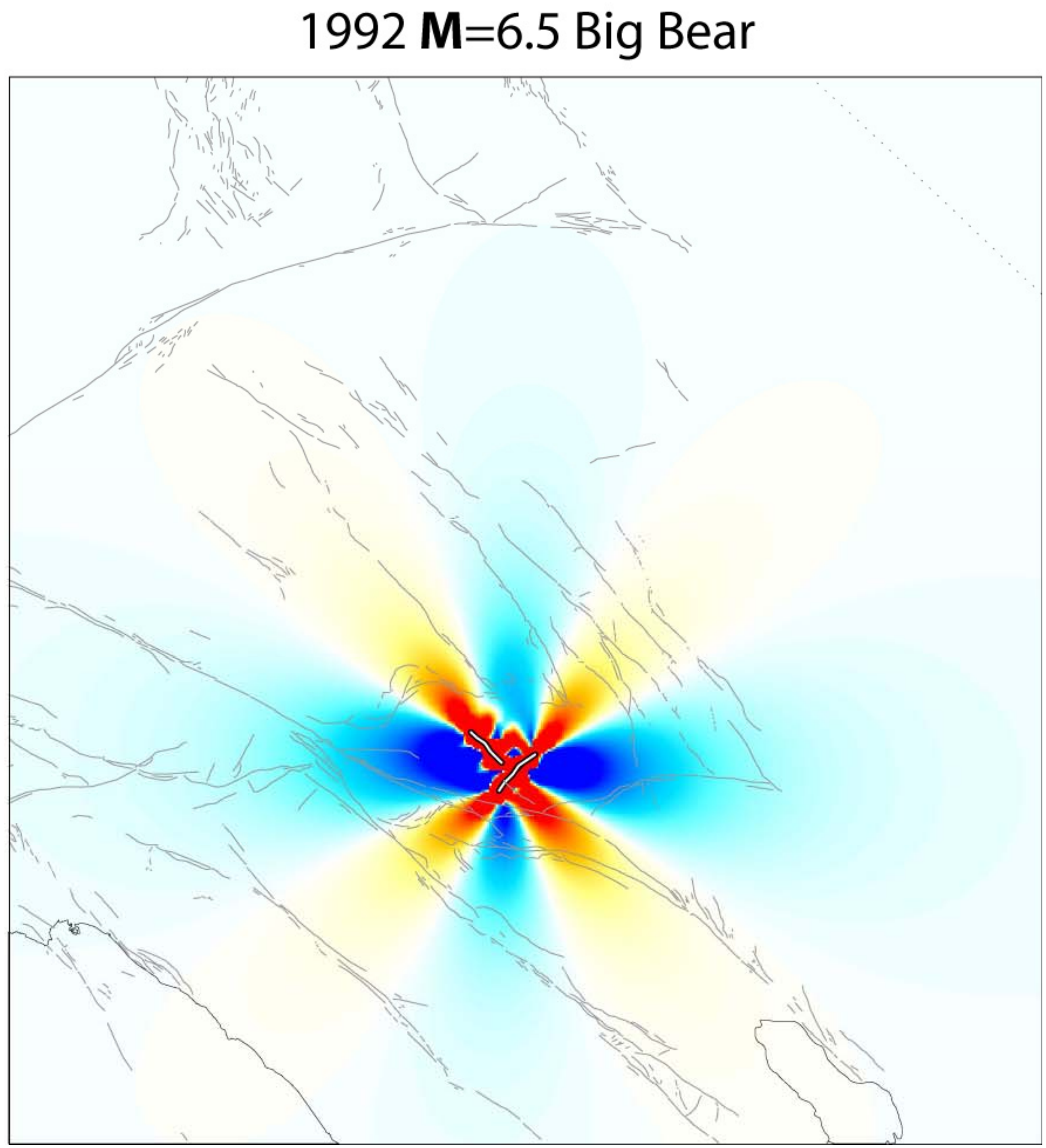
1992 **M**=7.4 Landers



from Todal et al (JGR, 2005)

Coulomb stress imparted by mainshocks

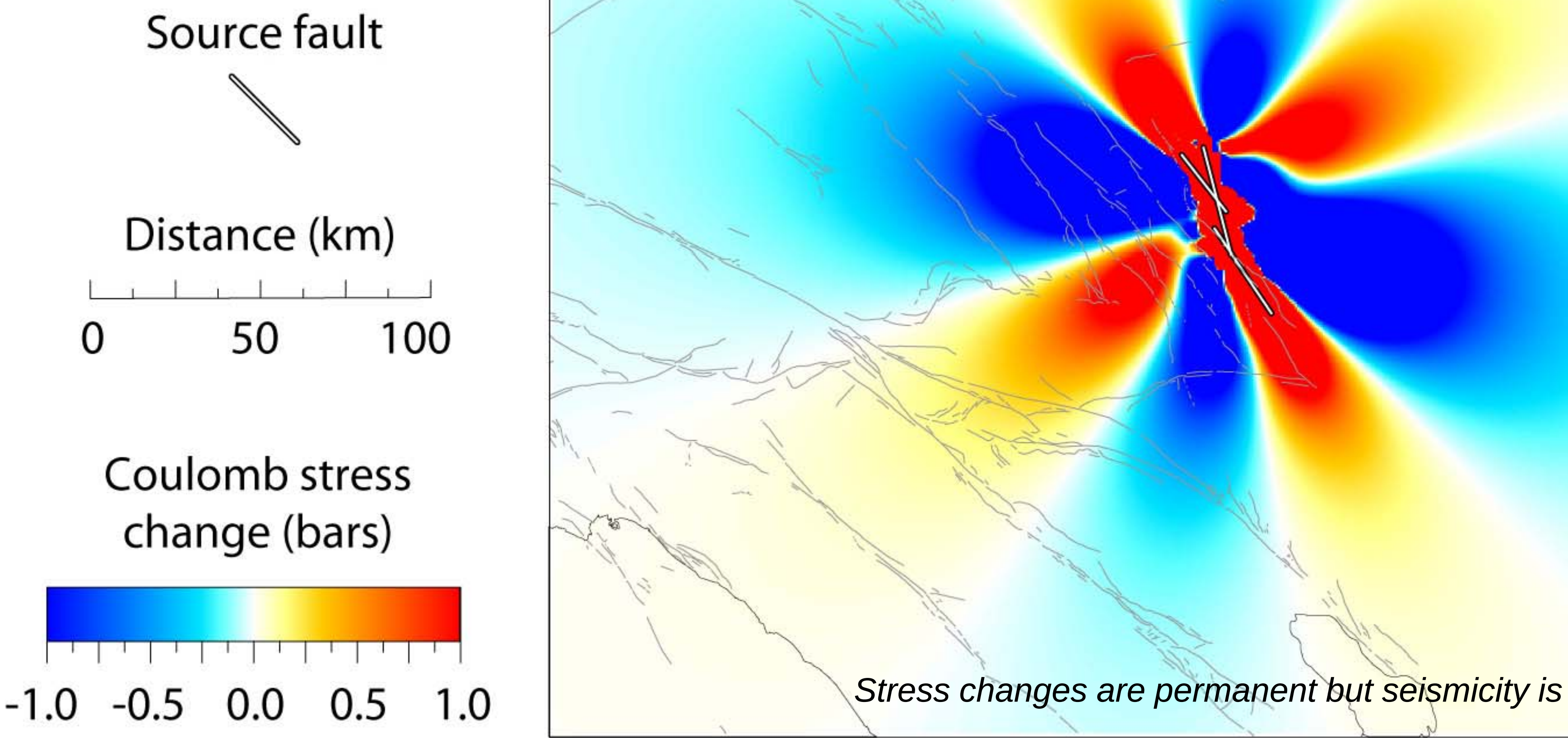




from Todal et al (JGR, 2005)

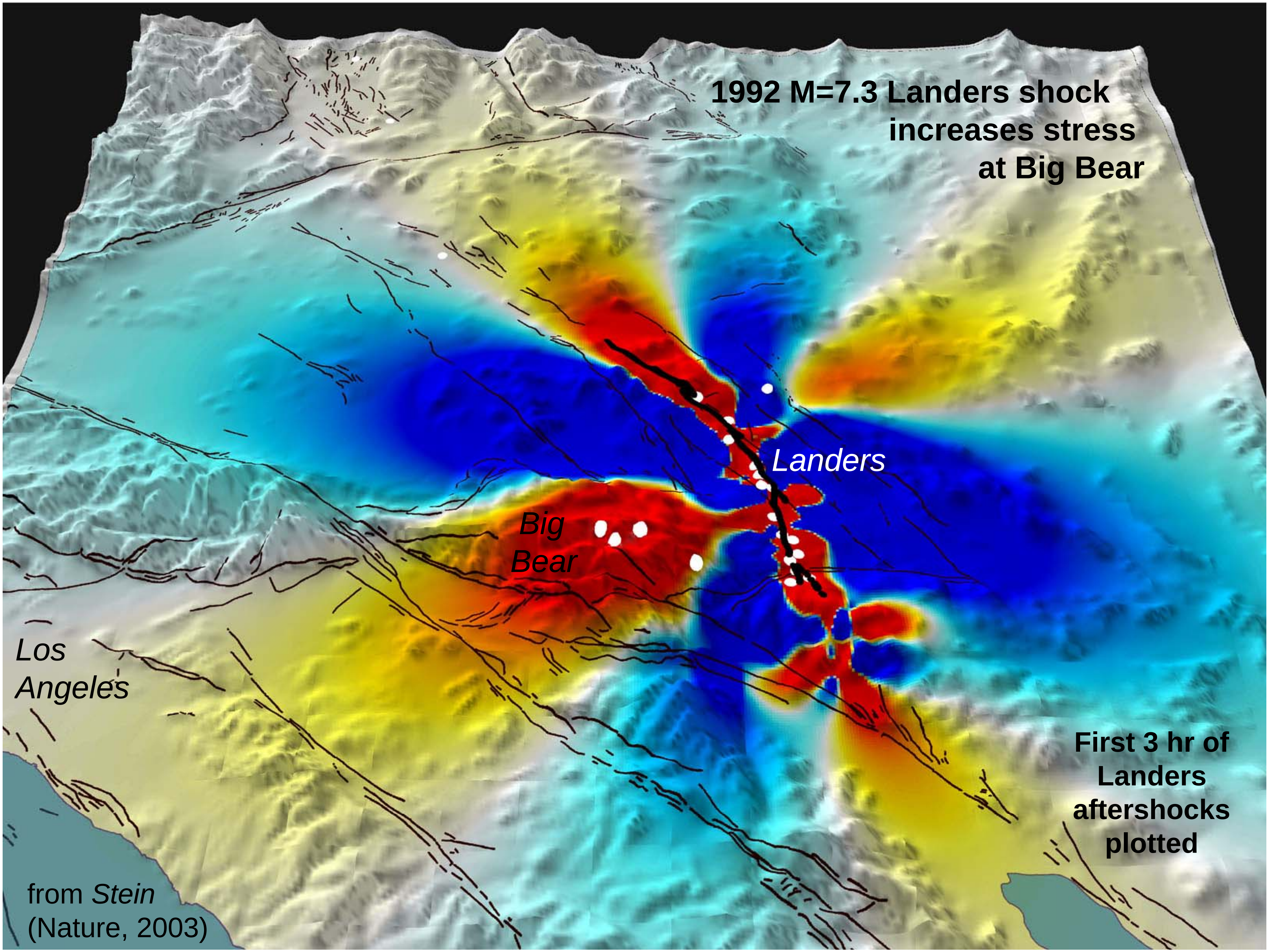
Coulomb stress imparted by mainshocks

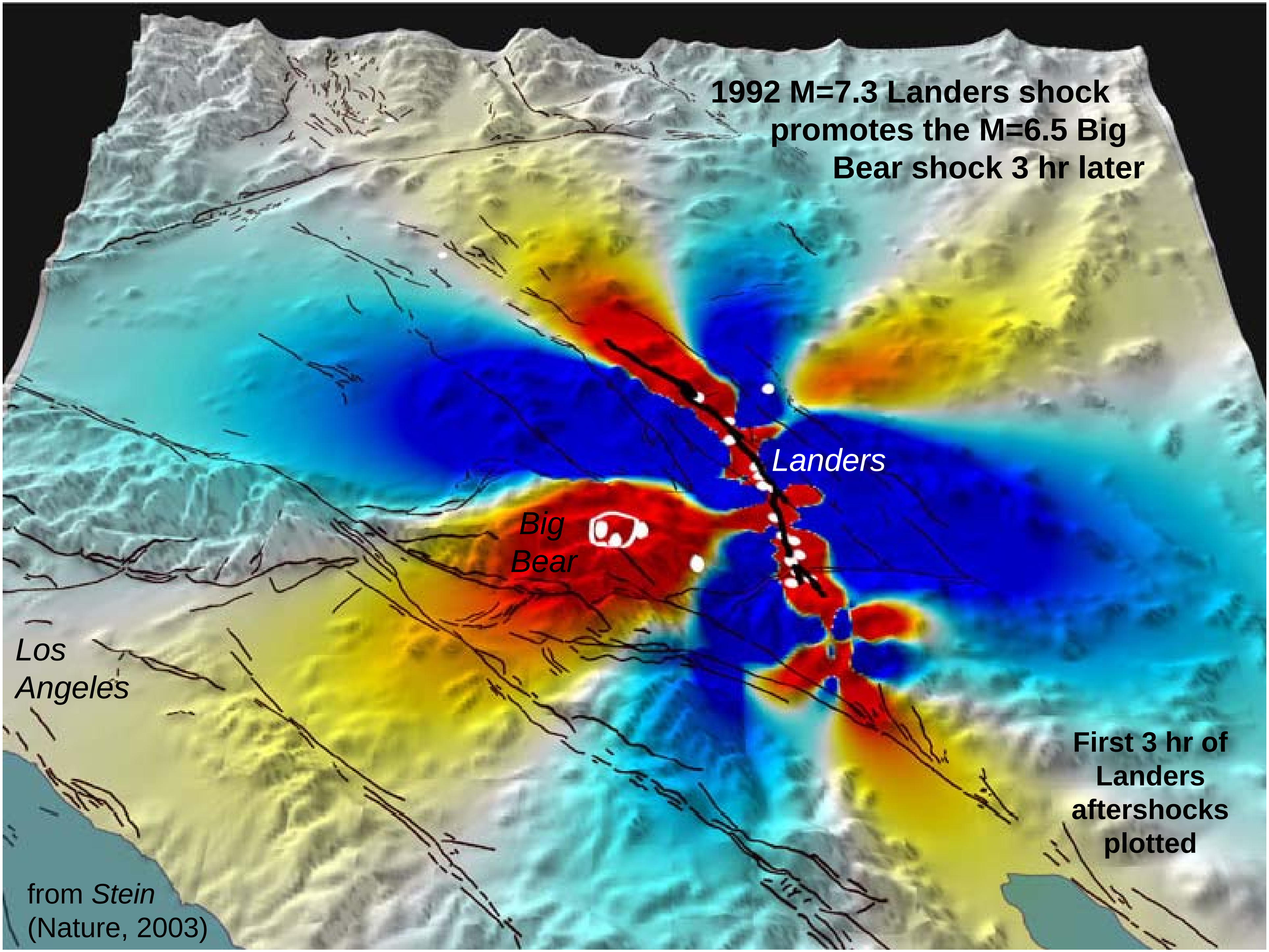
1999 **M**=7.1 Hector Mine

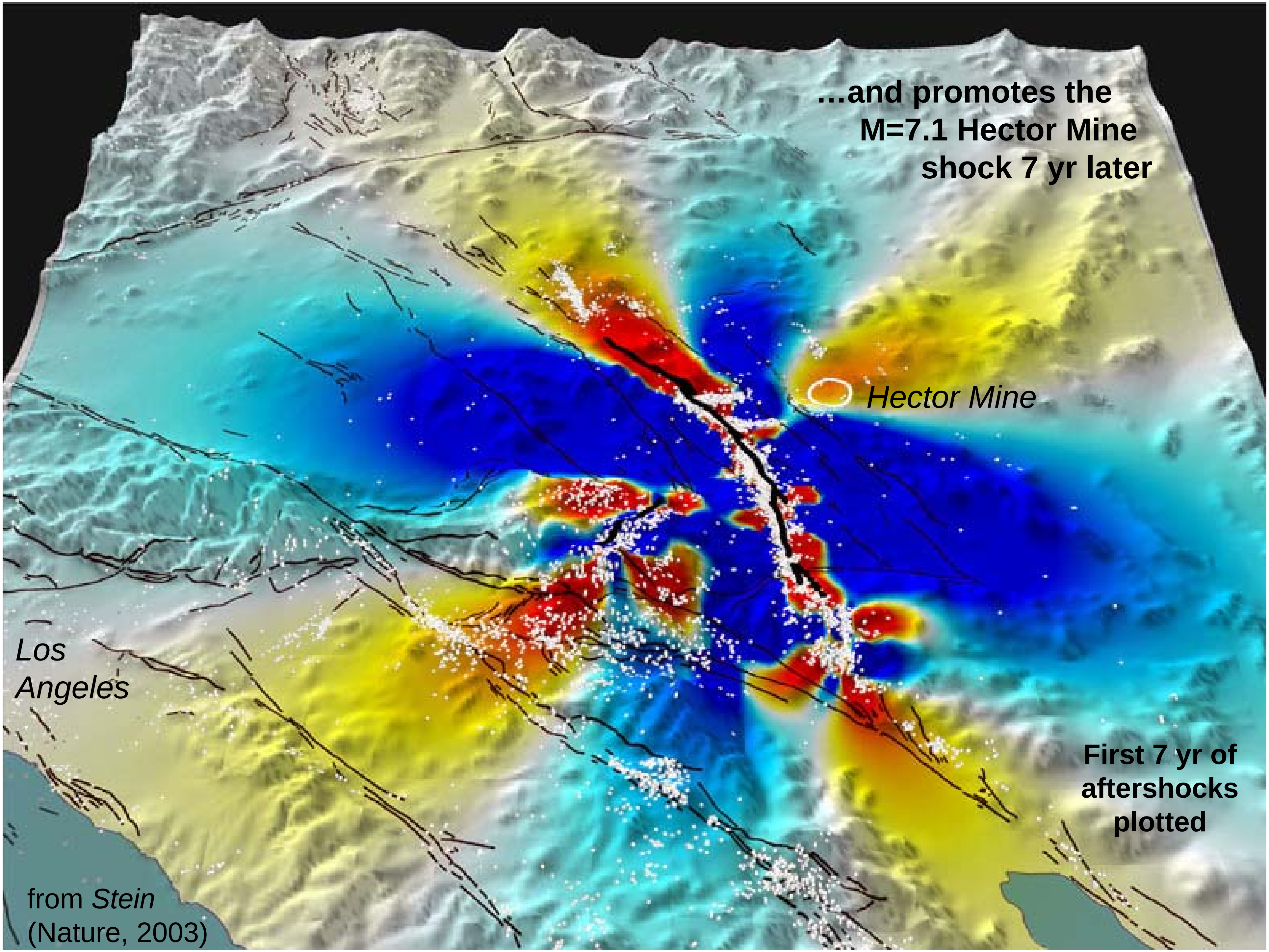


Stress changes are permanent but seismicity is not

from Todal et al (JGR, 2005)


1992 M=7.3 Landers shock
increases stress
at Big Bear
Landers
Big
Bear
Los
Angeles
First 3 hr of
Landers
aftershocks
plotted
from Stein
(Nature, 2003)


1992 M=7.3 Landers shock promotes the M=6.5 Big Bear shock 3 hr later
Landers
Big Bear
Los Angeles
First 3 hr of Landers aftershocks plotted
from *Stein* (Nature, 2003)



...and promotes the
M=7.1 Hector Mine
shock 7 yr later
Hector Mine
Los
Angeles
First 7 yr of
aftershocks
plotted
from *Stein*
(Nature, 2003)

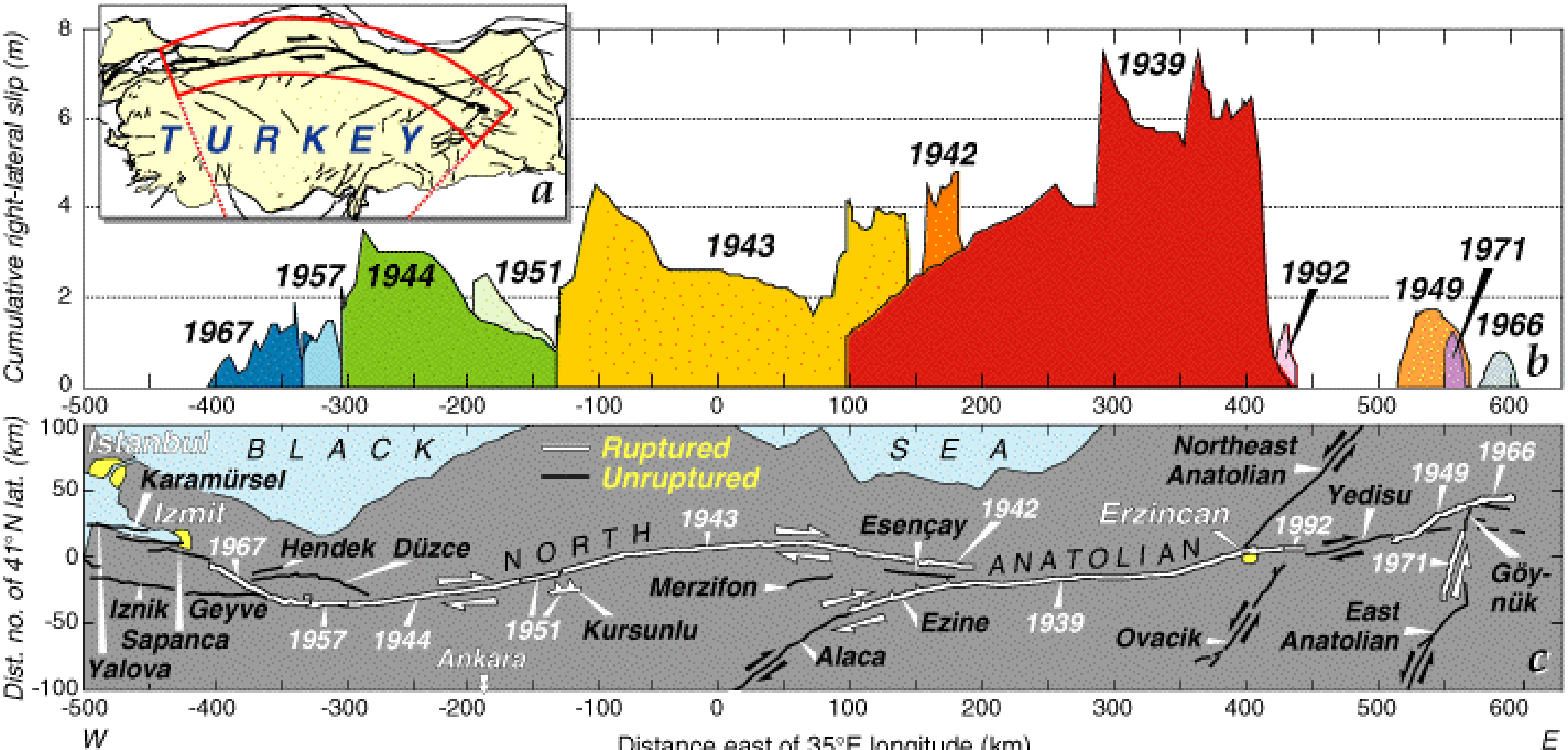


Figure 1 *17 Oct 96* **Stein et al.**

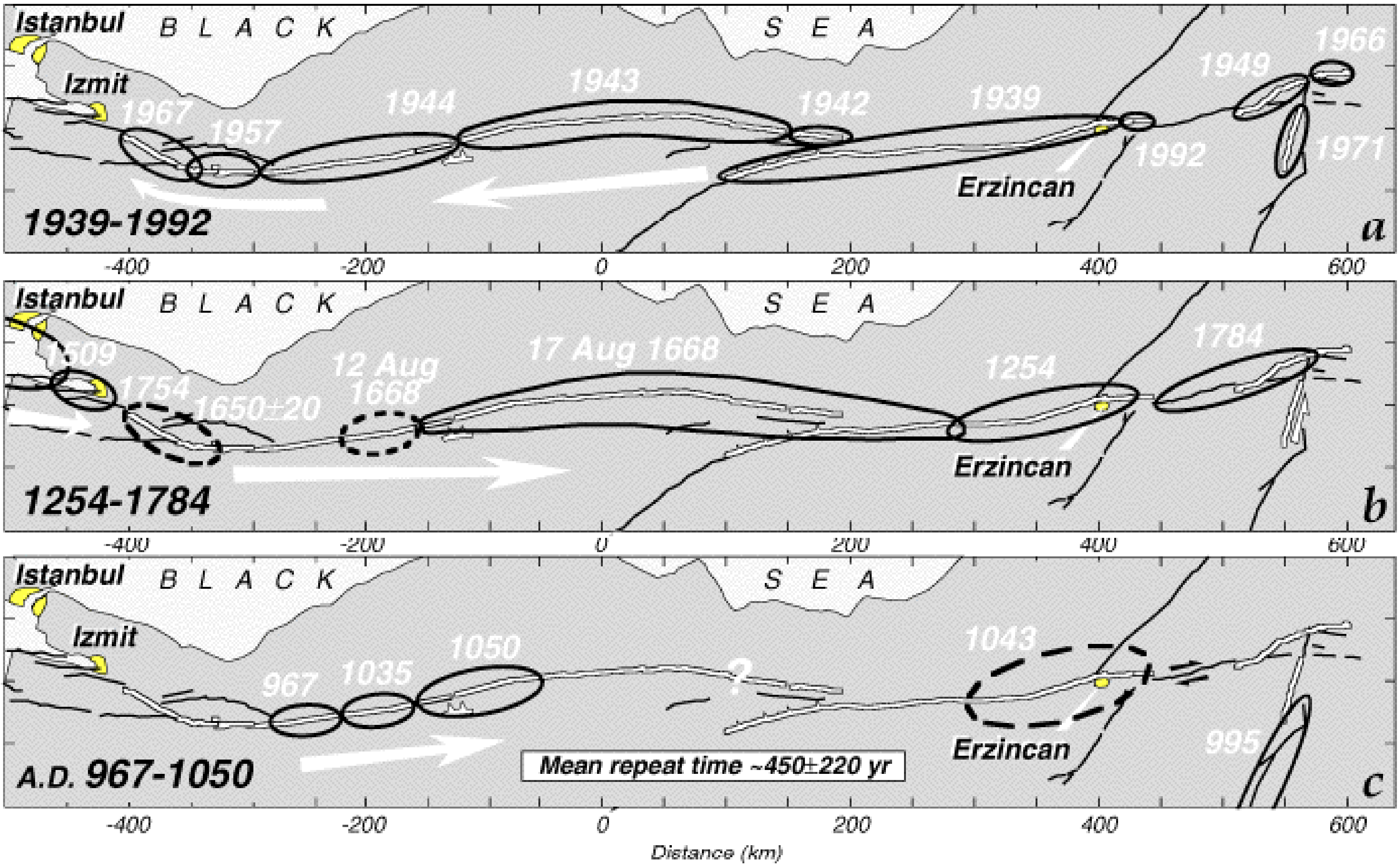
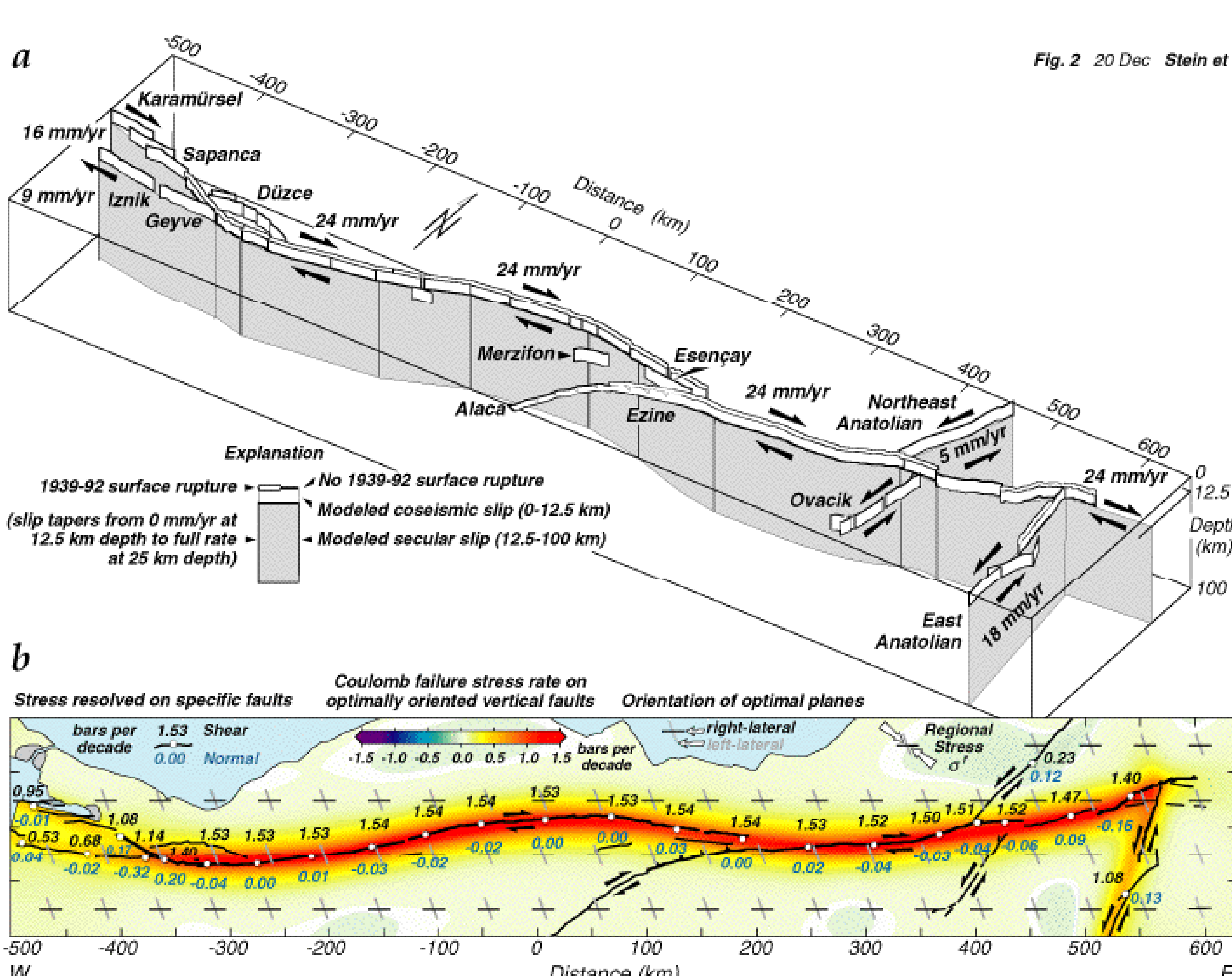


Fig. 7 17 Oct 96 **Stein et al.**


a
Karamürsel
16 mm/yr
Sapanca
9 mm/yr
Iznik
Geyve
Düzce
24 mm/yr
N
Distance (km)
24 mm/yr
Merzifon
Esençay
Alaca
Ezine
24 mm/yr
Northeast Anatolian
5 mm/yr
Ovacik
24 mm/yr
Depth (km)
East Anatolian
18 mm/yr
Explanation
1939-92 surface rupture
No 1939-92 surface rupture
Modeled coseismic slip (0-12.5 km)
(slip tapers from 0 mm/yr at 12.5 km depth to full rate at 25 km depth)
Modeled secular slip (12.5-100 km)
b
Stress resolved on specific faults
Coulomb failure stress rate on optimally oriented vertical faults
Orientation of optimal planes
bars per decade
1.53 Shear
0.00 Normal
-1.5 -1.0 -0.5 0.0 0.5 1.0 1.5
bars per decade
right-lateral
left-lateral
Regional Stress
Distance (km)
W
E

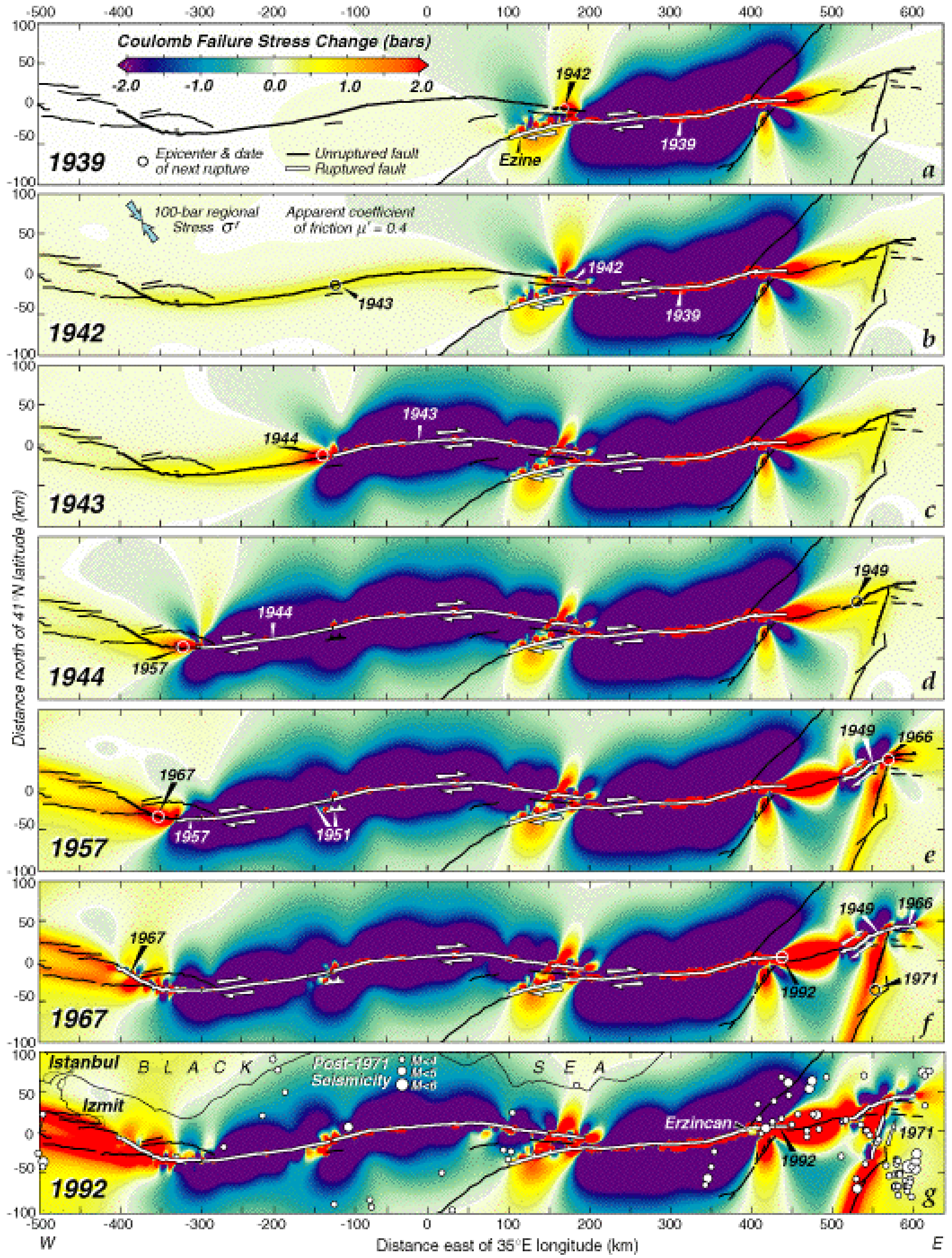
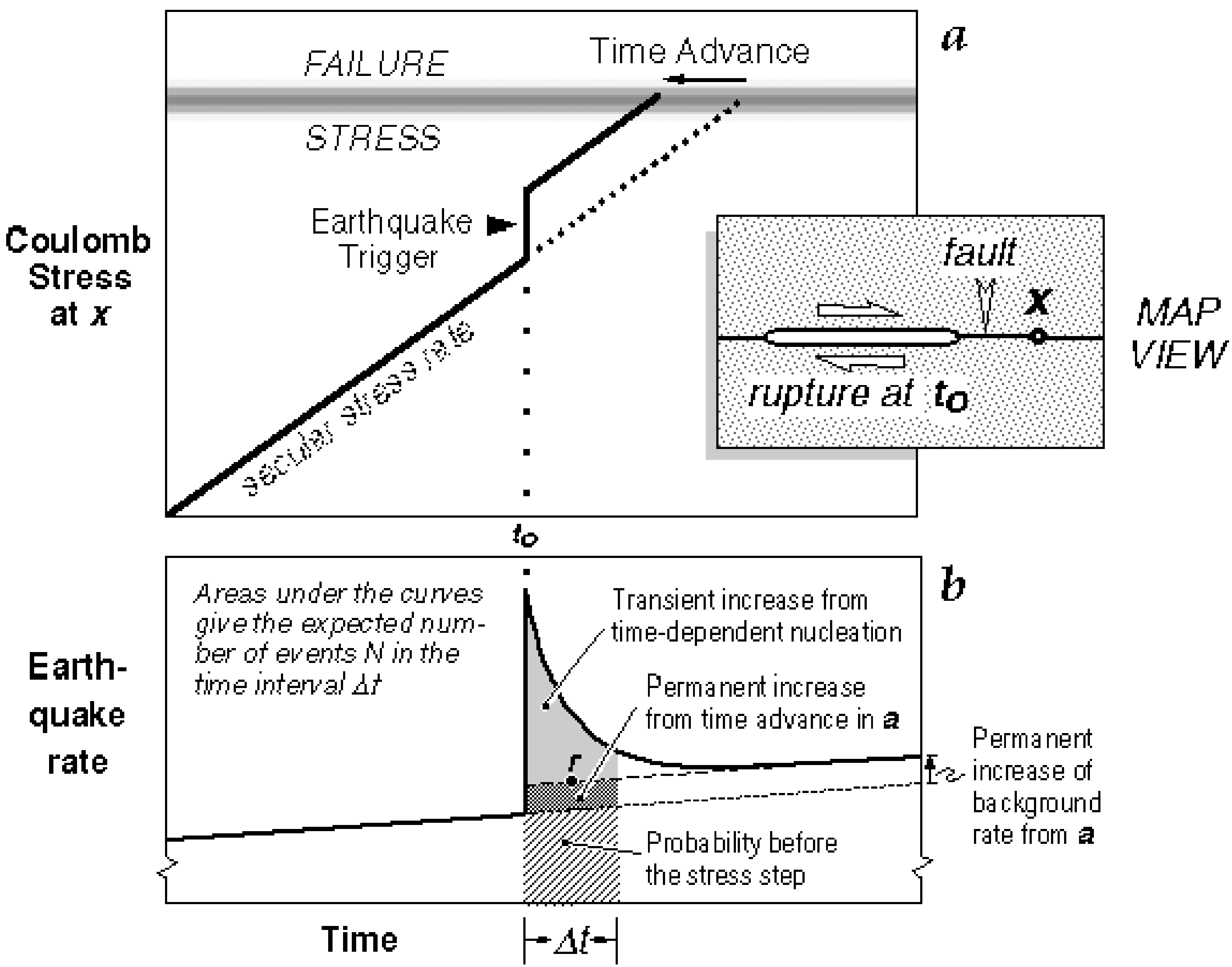


Figure 4 *17 Oct 96* **Stein et al.**

From Stress Change to Earthquake Probability Change



http://quake.usgs.gov/research/deformation/modeling/animations/index.html